HW 10 Solution

1) Shot-noise limited system

a) $R_b = 500\,Mb/s$, NRZ $\Rightarrow \Delta f = \frac{1}{\tau} = 500\,MHz$, $BER = 2\times10^{-11}$, $\lambda = 1.3\,\mu m$

$\eta = 0.9$, $I_d \simeq 0 \Rightarrow n_n = 0 \Rightarrow BER = e^{-n_s} \Rightarrow n_s = -\ln(BER) = 25$

(photo electrons, must be integer)

$$n_p = \frac{n_s}{\eta} = \frac{25}{0.9} = 28 \text{ (photon/bit)}$$

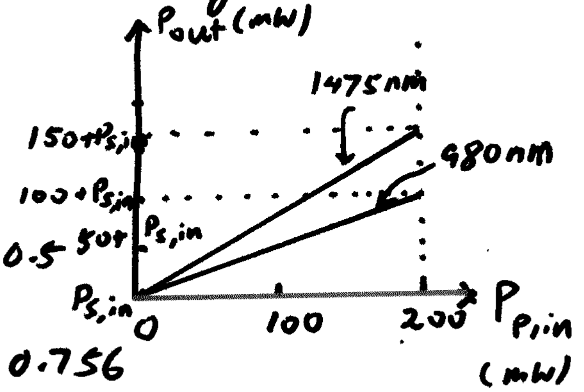
$$P_{in} = n_p E(J) * \frac{1}{\tau} = 28 \times \frac{1.24}{1.3} \times 1.6\times10^{-19} \times 5\times10^{8} = 2.137\times10^{-9}\,(W)$$

b) $n_p = 28$ photons / bit

c) The required power is much lower in the shot-noise limited case $\Rightarrow$ Req # of photons/bit is also very low.

d) Use APD. This is demonstrated by results from problems 1 and 3 in HW9. $M \gg 1 \Rightarrow SNR \simeq M^2(RP_{in})^2/\overline{i_s^2}$. Other possible way is to cool temperature to very low value on the level of mK.

2) a) $PCE = \dfrac{P_{out} - P_{s,in}}{P_{p,in}} \simeq \dfrac{P_{out}}{P_{p,in}} \leq \dfrac{\lambda_p}{\lambda_s}$



$PCE \leq \dfrac{980}{1545} = 0.6343$ Compare to 0.5

$PCE \leq \dfrac{1475}{1545} = 0.9547$ Compare to 0.756

These verify λ_p/λ_s give the max. PCE

b) $P_{out} = PCE \times P_{p,in} + P_{s,in}$

For 980 nm, $P_{out} = 0.5 P_{p,in} + P_{s,in}$

For 1475 nm, $P_{out} = 0.756\, P_{p,in} + P_{s,in}$

3) a) $G = (P_{out})_{dBm} - (P_{in})_{dBm} = 28\,dB = 10^{2.8} = 631$

(note: $P_{out} = 30\,dBm$, $P_{in} = 2\,dBm$ at 1582 nm)

b) Min pump power occurs when PCE is max. $\Rightarrow \lambda_p$ is the longest possible, i.e. $\lambda_p = 1480\,nm$.

$$1 + \frac{\lambda_p}{\lambda_s}\frac{P_{p,in}}{P_{s,in}} = G \Rightarrow P_{p,in} = (G-1)\frac{\lambda_s}{\lambda_p} P_{s,in} = (631-1)\frac{1582}{1480} 10^{0.2}\,(mW)$$

$\Rightarrow P_{p,in} = 1067\,(mW) = 1.067\,(W)$

4) $\eta = 0.6$, $R = 0.7\,(A/W)$, $P_{in} = 1\,(\mu W)$, $\lambda = 1560\,nm$, $\Delta\nu_{opt} = 3.77\times10^{12}\,(Hz)$

$\Delta f = 10^9 \times 2\,(Hz)$, $n_{sp} = 2$, $R_L = 1000\,(\Omega)$.

a) $G = 28\,dB = 10^{2.8} = 631$ (Assume $T = 300k$)

$$\overline{i_T^2} = \frac{4kT\Delta f}{R_L} = \frac{4\times1.38\times10^{-23}\times300\times2\times10^9}{1000} = 3.312\times10^{-14}\,(A^2)$$

$\overline{i_S^2} = 2q(RGP_{in}+I_d)\Delta f = 2\times1.6\times10^{-19}(0.7\times631\times10^{-6})\times2\times10^9 = 2.827\times10^{-13}\,(A^2)$

$\overline{i_{shot-ASE}} = 2qRP_{ASE}\Delta f = 2qRS_{ASE}\Delta\nu_{opt}\Delta f$, $S_{ASE} = h\nu n_{sp}(G-1)$

$= 2\times1.6\times10^{-19}\times6.841\times10^{-4}\times2\times10^9\times0.7$ $\quad = \frac{1.24}{1.56}\times1.6\times10^{-19}\times2\times630$

$= 2.706\times10^{-13}\,(A^2)$ $\quad = 1.602\times10^{-16}\,(W/Hz)$

$\overline{i^2_{sig-ASE}} = 2(2RGP_{in})RS_{ASE}\Delta f = 2(2\times0.7\times631\times10^{-6})0.7\times1.602\times10^{-16}\times2\times10^9$

$= 3.964\times10^{-10}\,(A^2)$

$\overline{i^2_{ASE-ASE}} = R^2S^2_{ASE}(2\Delta\nu_{opt}-\Delta f)\Delta f = 0.7^2(1.602\times10^{-16})^2(2\times3.77\times10^{12}-2\times10^9)2\times10^9$

$= 1.897\times10^{-10}\,(A^2)$

$G = 38\,dB = 10^{3.8} = 6310$, $\overline{i_T^2} = 3.312\times10^{-10}\,(A^2)$ Remain the same

$S_{ASE} = h\nu(G-1)n_{sp} = \frac{1.24}{1.56}\times1.6\times10^{-19}\times6309\times2 = 1.605\times10^{-15}\,(W/Hz)$

$\overline{i_S^2} = 2.827\times10^{-12}\,(A^2)$, $\overline{i^2_{shot-ASE}} = 2.711\times10^{-12}\,(A^2)$

$\overline{i^2_{sig-ASE}} = 3.969\times10^{-8}\,(A^2)$, $\overline{i^2_{ASE-ASE}} = 1.903\times10^{-8}\,(A^2)$

b) $\Delta\nu_{opt} = 1.25\times10^{11}\,Hz$ $\quad G = 25\,dB = 10^{2.5} = 316.2$

$\overline{i_T^2} = 3.312\times10^{-14}\,(A^2)$ (same) $\quad \overline{i_S^2} = 1.417\times10^{-13}\,(A^2)$

$S_{ASE} = \frac{1.24}{1.56}\times1.6\times10^{-19}\times2\times316.2 = 8.019\times10^{-17}\,(W/Hz)$

$\overline{i^2_{sig-ASE}} = 9.939\times10^{-11}\,(A^2)$, $\overline{i^2_{shot-ASE}} = 4.489\times10^{-15}\,(A^2)$

$\overline{i^2_{ASE-ASE}} = 0.7^2(8.019\times10^{-17})^2(2\times1.25\times10^{11}-2\times10^9)2\times10^9 = 1.563\times10^{-12}\,(A^2)$

$G = 35\,dB = 10^{3.5} = 3162$. $\overline{i_T^2}$ remains the same. $\Delta\nu_{opt} = 1.25\times10^{11}\,Hz$

$\overline{i_S^2} = 1.417\times10^{-12}\,(A^2)$, $\overline{i^2_{shot-ASE}} = 4.503\times10^{-14}\,(A^2)$

$\overline{i^2_{sig-ASE}} = 4.775\times10^{-9}\,(A^2)$, $\overline{i^2_{ASE-ASE}} = 1.571\times10^{-10}\,(A^2)$

Notice that optical filter lets $\overline{i^2_{sig-ASE}}$ dominate.

Without optical filter, $\overline{i^2_{ASE-ASE}}$ and $\overline{i^2_{sig-ASE}}$ are of the same order.

5) EDFA $G = 22\,dB = 10^{2.2} = 158.5$, $R = 0.7\,A/W$, $NEP = 5\times10^{-11}\,W/\sqrt{Hz}$, $n_{sp} = 8$

$\lambda = 1.54\,\mu m$

NEP is dominated by thermal noise $\Rightarrow NEP = \sqrt{\overline{i_T^2}}/(R\sqrt{\Delta f})$

$$\overline{i^2}_{sig\text{-}ASE} = 4 \mathcal{R} P_{in} G S_{ASE} \mathcal{R} \cancel{\Delta f} \le \overline{i^2_T} = (NEP\,\mathcal{R})^2 \cancel{\Delta f}$$

$$\Rightarrow P_{in} \le \frac{NEP^2}{4G\,S_{ASE}} = \frac{(5\times10^{-11})^2}{4\times 158.5 \times \frac{1.24}{1.54} \times 1.6\times10^{-19} \times 8\,(158.5-1)} = 2.429\times10^{-8}\,(W)$$

6) a) $\phi_{ijk} = \frac{2\pi}{\lambda}\left[n_1 R\,(2-(i+j)k\alpha^2) + n_2 k \Delta L\right]$

$$\phi_{ijk-1} = \frac{2\pi}{\lambda}\left[n_1 R\,(2-(i+j)(k-1)\alpha^2) + n_2 (k-1) \Delta L\right]$$

$$\phi_{ijk} - \phi_{ijk-1} = 2\pi$$

$$\Rightarrow \frac{2\pi}{\lambda}\left[n_2 \Delta L - (i+j)\, n_1 R \alpha^2\right] = 2\pi \Rightarrow \lambda = n_2 \Delta L - (i+j)\, n_1 R \alpha^2$$

If $\lambda_\phi = n_2 \Delta L$ & $\Delta\lambda = n_1 \alpha^2 R$, then $\lambda = \lambda_\phi - (i+j)\Delta\lambda$

b) This design is just one possibility. There are other possible answers.

$n_1 = n_2 = 1.5$, $\Delta L = 1.033\,\mu m$, $R = 10\,mm$,

$\alpha = 5\times10^{-4} \Rightarrow \Delta\lambda = 1.5\times(5\times10^{-4})^2\, 10^{-2} = 3.75\,nm$

$\Rightarrow \lambda_\phi = n_2 \Delta L = 1.5 \times 1.033 = 1549.5\,nm$

$\lambda_{14} = 1549.5 - 14\times 3.75 = 1497\,nm$

$\lambda_{10} = 1512\,nm$, $\lambda_9 = 1515\,nm$