HW1 Solution

1) $\vec{h} = \hat{z}5\cos(10^{11}t + kx)$(A/m), i.e. $\hat{H} = \hat{z}$. $H_o = 5$(A/m)

a) $\omega = 10^{11}$ rad/s

b) $k = \dfrac{\omega}{c} = \dfrac{10^{11}}{3\times10^{8}} = 333.3$(rad/m) Note: Medium is air $\Rightarrow u_p = c$

c) Propagation direction is –x.

d) Polarization direction is y. Note: Polarity is not important for polarization.

e) $\vec{E} = \hat{E}E_o e^{jkx}$ in phasor form. In terms of time varying function, $\vec{e} = \hat{E}\,|E_o|\cos(\omega t + kx + \phi_E)$. Notice that $E_o = |E_o|\angle\phi_E$ is complex.

According to right hand rule: $\hat{E} = \hat{H}\times\hat{k} = \hat{z}\times -\hat{x} = -\hat{y}$.

By Ohm's law, $E_o\,(\text{V/m}) = \eta(\Omega)\,H_o\,(\text{A/m}) \Rightarrow E_o = \eta_o H_o = 120\pi\times5 = 600\pi$(V/m)

Therefore, $\vec{e} = -\hat{y}600\pi\cos(10^{11}t + 333.3x)$(V/m)

3) a) $\Delta\nu = \dfrac{\nu}{\lambda_0}\Delta\lambda = \dfrac{c}{\lambda_0^2}\Delta\lambda = \dfrac{3\times10^{8}}{(1.5\times10^{-6})^2}\times3\times10^{-9} = 4\times10^{11}$(Hz)

$\Delta\nu_{total} = N\Delta\nu = 75\times4\times10^{11} = 3\times10^{13}$(Hz) where N is number of channels.

b) $\Delta\nu' = \dfrac{\nu}{\lambda_0'}\Delta\lambda = \dfrac{c}{\lambda_0'^2}\Delta\lambda = \dfrac{3\times10^{8}}{(0.5\times10^{-6})^2}\times3\times10^{-9} = 3.6\times10^{12}$(Hz)

$N = \Delta\nu_{total}/\Delta\nu' = \dfrac{3\times10^{13}}{3.6\times10^{12}} \approx 8$. Note: Just take the integer part.

4) a) $E(\text{eV}) = \dfrac{h\nu}{q}\dfrac{1}{\lambda} = \dfrac{1.24}{\lambda(\mu m)} = 0.8(\text{eV}) \Rightarrow \lambda = \dfrac{1.24}{0.8} = 1.55(\mu\text{m})$

$f = \dfrac{c}{\lambda} = \dfrac{3\times10^{8}}{1.55\times10^{-6}} = 1.935\times10^{14}$(Hz)

b) $\eta_{eo} = \dfrac{1}{8}$ (8 electrons for 1 photon). Let number of electrons = N_e, number of photons = N_p.

$N_e = I/q$ and $N_p = \eta_{eo}N_e = \eta_{eo}I/q$.

Optical power $= qE(eV)\times N_p = E(eV)\times\eta_{eo}I = 0.8\times2.5/8 = 0.25(W)$

Extra-credit

a)

$\vec{k} = -2\hat{y} + 4\hat{z}, k = |\vec{k}| = \sqrt{4+16} = 4.47(rad/m); \omega = kc = 1.342\times10^{9}(rad/s); \lambda = 2\pi/k = 1.405(m)$

b) $\vec{H} = (10\hat{y} + 5\hat{z})e^{-j(-2y+4z)}$

c) $E_o = H_o\eta_o; \hat{E} = \hat{H}\times\hat{k} = \dfrac{10\hat{y}+5\hat{z}}{H_o}\times\dfrac{(-2\hat{y}+4\hat{z})}{4.47} = \dfrac{10\hat{x}+40\hat{x}}{4.47H_o}; \vec{E} = 4217\hat{x}e^{-j(-2y+4z)}(V/m)$

$\vec{e} = 4217\hat{x}\cos(1.342\times10^{9}t + 2y - 4z)(V/m)$

d) $\vec{S}_{ave} = \hat{k}0.5|E_o|^2/\eta_o = \dfrac{(-2\hat{y}+4\hat{z})}{4.47}\times0.5|4217|^2/377 = 1.055\times10^{4}(-\hat{y}+2\hat{z})(W/m^2)$