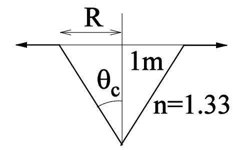
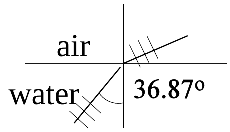
HW 2 Solution

1) a) The light circle must have the cone angle within the critical angle between water and air, i.e. $\theta_c = \sin^{-1}(1/1.333) = 48.59^\circ$ and radius of circle R = $1 \times \tan 48.59^\circ = 1.134(m)$.



b) This a Brewster angle problem which requires p-polarization. The Brewster angle $\theta_B = \tan^{-1}(1/1.333) = 36.87^\circ$



c) $T = 1 - \left(\dfrac{1.333-1}{1.333+1}\right)^2 = \dfrac{48}{49}$

2) $w_o = 100\mu m, \lambda_o = 630nm \Rightarrow z_o = \pi w_o^2 / \lambda_o = \pi 10^{-8}/(630\times 10^{-9}) = 0.04987(m)$

a) At z=1m, $w = w_o\sqrt{1+(z/z_o)^2} = 10^{-4}\sqrt{1+(1/0.04987)^2} = 2.008(mm)$. b) At z=1cm, w=102(μm)

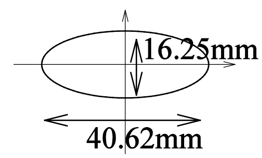
c) $\theta = 2\lambda_o/(\pi w_o) = 4.011(mrad)$ which is not valid for $-z_o = -0.04987 < z < z_o = 0.04987$(m).

3) $w_{ox} = 0.05mm, w_{oy} = 0.125mm, \lambda_o = 638nm, z = 5m$

$z_{ox} = \pi w_{ox}^2/\lambda_o = \pi(0.05\times 10^{-3})^2/(638\times 10^{-9}) = 0.01231(m)$

$z_{oy} = \pi w_{oy}^2/\lambda_o = 0.07694(m)$

$w_x = w_{ox}\sqrt{1+(z/z_{ox})^2} = 20.31(mm)$, $w_y = w_{oy}\sqrt{1+(z/z_{oy})^2} = 8.124(mm)$



4) Mach-Zehnder: Optical path for beam 2 = $2d_2 + n_1 d_1$, optical path for beam 1 = d_1

Constructive interference condition: $k_o(2d_2 + n_1 d_1 - d_1) = 2\pi m \Rightarrow 2d_2 + n_1 d_1 - d_1 = m\lambda_o$

Michelson: Optical path for beam 1 = $commom + 2L_1$, optical path for beam 2 = $common + 2n_2 L_2$

Constructive interference condition: $2k_o(L_2 n_2 - L_1) = 2\pi m \Rightarrow L_2 n_2 - L_1 = m\lambda_o/2$, where m is an integer

5) $E_{oi} + E_{or} = E_{ot}$ —(1), $(E_{oi} - E_{or})\dfrac{\cos\theta_1}{\eta_1} = E_{ot}\dfrac{\cos\theta_2}{\eta_2} \Rightarrow (E_{oi} - E_{or})n_1\cos\theta_1 = E_{ot}n_2\cos\theta_2$—(2)

(2)/(1):

$$n_2\cos\theta_2 = n_1\cos\theta_1\frac{E_{oi}-E_{or}}{E_{oi}+E_{or}} \Rightarrow \frac{E_{oi}-E_{or}}{E_{oi}+E_{or}} = \frac{n_2\cos\theta_2}{n_1\cos\theta_1} \Rightarrow \frac{E_{oi}+E_{or}-(E_{oi}-E_{or})}{E_{oi}+E_{or}+(E_{oi}-E_{or})} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

$$\Rightarrow \Gamma_\perp = \frac{E_{or}}{E_{oi}} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

Extra-Credit

$\cos\theta_1(E_{oi} + E_{or}) = \cos\theta_2 E_{ot}$ —(3), $\dfrac{1}{\eta_1}(E_{oi} - E_{or}) = \dfrac{1}{\eta_2}E_{ot} \Rightarrow n_1(E_{ot} - E_{or}) = n_2 E_{ot}$ —(4)

(3)/(4):

$$\frac{n_1(E_{oi}-E_{or})}{\cos\theta_1(E_{oi}+E_{or})} = \frac{n_2}{\cos\theta_2} \Rightarrow \frac{E_{oi}-E_{or}}{E_{oi}+E_{or}} = \frac{n_2\cos\theta_1}{n_1\cos\theta_2} \Rightarrow \frac{E_{oi}+E_{or}-(E_{oi}-E_{or})}{E_{oi}+E_{or}+(E_{oi}-E_{or})} = \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1}$$

$$\Rightarrow \Gamma_\parallel = \frac{E_{or}}{E_{oi}} = \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1}$$

Extra-credit

6) a) Apply Snell's law $n_1\sin\theta_1 = n_2\sin\theta_2 \Rightarrow n_1/n_2 = \sin\theta_2/\sin\theta_1$ to $\Gamma_\perp$ and $\Gamma_\parallel$:

$$\Gamma_\perp = \frac{E_{or}}{E_{oi}} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{\cos\theta_1 - \dfrac{n_2}{n_1}\cos\theta_2}{\cos\theta_1 + \dfrac{n_2}{n_1}\cos\theta_2} = \frac{\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2}{\sin\theta_2\cos\theta_1 + \sin\theta_1\cos\theta_2} = \frac{-\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}$$

$$\Gamma_{\|} = \frac{E_{or}}{E_{oi}} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1} = \frac{\sin\theta_2 \cos\theta_2 - \sin\theta_1 \cos\theta_1}{\sin\theta_2 \cos\theta_2 + \sin\theta_1 \cos\theta_1} = \frac{\sin 2\theta_2 - \sin 2\theta_1}{\sin 2\theta_2 + \sin 2\theta_1}$$

$$= \frac{2\sin\left(\frac{2(\theta_2 - \theta_1)}{2}\right)\cos\left(\frac{2(\theta_2 - \theta_1)}{2}\right)}{2\sin\left(\frac{2(\theta_2 + \theta_1)}{2}\right)\sin\left(\frac{2(\theta_2 - \theta_1)}{2}\right)} = \frac{\tan(\theta_2 - \theta_1)}{\tan(\theta_2 + \theta_1)} = -\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_2 + \theta_1)}$$

b) For plotting of $|\tau_{\perp}|^2$ and $|\tau_{\|}|^2$, see the mathcad program and printout on the last page. Notice that they are not exactly equal to T owing to mismatch of refractive indices at the boundary.

$$\theta_1 := 0, 0.01 .. \frac{\pi}{2} \qquad n_1 := 3.6 \qquad n_2 := 1 \qquad \cos\theta_2(\theta_1) := \sqrt{1 - \left(\frac{n_1}{n_2}\cdot\sin(\theta_1)\right)^2}$$

$$\Gamma_p(\theta_1) := \frac{(n_1\cdot\cos\theta_2(\theta_1) - n_2\cdot\cos(\theta_1))}{(n_1\cdot\cos\theta_2(\theta_1) + n_2\cdot\cos(\theta_1))}$$

$$\Gamma_s(\theta_1) := \frac{(-n_2\cdot\cos\theta_2(\theta_1) + n_1\cdot\cos(\theta_1))}{(n_2\cdot\cos\theta_2(\theta_1) + n_1\cdot\cos(\theta_1))}$$

$$\tau_p(\theta_1) := n_1\cdot\frac{(1 - \Gamma_p(\theta_1))}{n_2} \qquad \tau_s(\theta_1) := 1 + \Gamma_s(\theta_1)$$