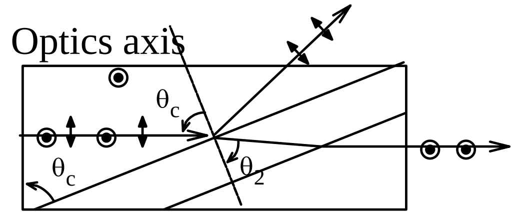
HW 3 Solution

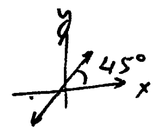
1) Notice that $n_{\|}$ is the index for electric field parallel to optic axis and $n_{\perp}$ is the index for electric field perpendicular to the optic axis. They have no connection with polarization (electric field direction) of the wave or the boundaries between materials, e.g. air and glass etc. Since $n_{\perp} > n_m > n_{\|}$ (where n_m is the index of the Canada balsam) and for totally internal refraction $n_{in} > n_{out}$, only the electric field perpendicular to the optic axis will satisfy the condition, i.e. p-polarized wave (TM) will be totally internal reflected according to the diagram. The critical angle in this case is:



$$\sin\theta_c = n_{out}/n_{in} \Rightarrow \theta_c = \sin^{-1}(n_m/n_{\perp}) = 68.25°$$

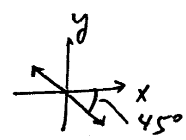
The s-polarized wave (TE) will transmit to the Canada balsam with an angle of θ_2. Apply Snell's law again, $n_{\|}\sin\theta_c = n_m \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}(n_{\|}\sin\theta_c/n_m) = \sin^{-1}(n_{\|}n_m/(n_m n_{\perp})) = 63.67°$

2) Input wave $\cos(\omega t)\hat{a}_x + \cos(\omega t)\hat{a}_y$



2 $\lambda/4$ plate in y will introduce π phase shift along y, i.e.

$$\cos(\omega t)\hat{a}_x + \cos(\omega t+\pi)\hat{a}_y$$
$$= \cos(\omega t)\hat{a}_x - \cos(\omega t)\hat{a}_y$$



Just like a $\lambda/2$ plate causinge rotation in polarization by $2\theta = 2\times 45° = 90°$.

1 $\lambda/4$ plate along x, 1 $\lambda/4$ plate along y.

$\Rightarrow \cos(\omega t + \frac{\pi}{2})\hat{a}_x + \cos(\omega t + \frac{\pi}{2})\hat{a}_y$

$\Rightarrow -\sin(\omega t)\hat{a}_x - \sin(\omega t)\hat{a}_y$

No change in amplitude ratio & relative phase $\Rightarrow$ just keep the original polarization.

3) a) $\Delta\phi = (k_{\perp} - k_{\|})d$ and for circularly polarized wave we need a quarter wave plate, i.e.

$\Delta\phi = \frac{2m+1}{2}\pi$. Hence, $d = \frac{(2m+1)\pi}{2(k_{\perp}-k_{\|})} = \frac{(2m+1)\lambda_o}{4(n_{\perp}-n_{\|})} = \frac{(2m+1)633\times10^{-9}}{4(1.658-1.486)} = (2m+1)\times 920\times10^{-9}(m)$

b) For a rotator, rotation angle $\theta = VHd \Rightarrow H = \frac{\theta}{Vd} = \frac{45}{2.64\times10^{-4}100} = 1.705(kA/m)$.

4)

To have constructive interference

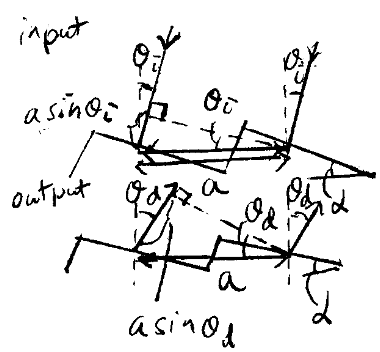
$k\Delta r = 2m\pi$ & $\Delta r = a\sin\theta_i + a\sin\theta_d$

m ← integer

$\Rightarrow \frac{\pi}{\lambda_o} a(\sin\theta_i + \sin\theta_d) = m\pi \Rightarrow a(\sin\theta_i + \sin\theta_d) = m\lambda_o$

Maximum diffraction when $\theta_i = \alpha \Rightarrow \theta_d = \alpha$

Therefore, $2a\sin\alpha = m\lambda_o \Rightarrow m\frac{\lambda_o}{a} = \sin\alpha$



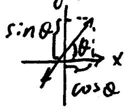
Extra-Credit

$$\vec{E} = \frac{e^{j\theta}}{\sqrt{2}}\hat{e}_R + \frac{e^{-j\theta}}{\sqrt{2}}\hat{e}_L = \frac{e^{j\theta}}{\sqrt{2}}\left(\frac{\hat{x}-j\hat{y}}{\sqrt{2}}\right) + \frac{e^{-j\theta}}{\sqrt{2}}\left(\frac{\hat{x}+j\hat{y}}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left[\hat{x}\left(e^{j\theta}+e^{-j\theta}\right) + \hat{y}\left(j e^{-j\theta} - j e^{j\theta}\right)\right]$$

real vector $\Rightarrow$ linear polarization $\longrightarrow$ $\hat{x}\cos\theta + \hat{y}\sin\theta$ //

$$\left[\text{note: } \frac{e^{j\theta}-e^{-j\theta}}{2j} = \sin\theta \qquad \frac{e^{j\theta}+e^{-j\theta}}{2} = \cos\theta\right]$$



Time varying form:

$$\vec{e} = \hat{x}\cos\theta\cos\omega t + \hat{y}\sin\theta\cos\omega t \ //$$

$$\vec{E} = \frac{e^{j\theta}}{\sqrt{2}} e^{j\theta_-}\hat{e}_R + \frac{e^{-j\theta}}{\sqrt{2}} e^{j\theta_+}\hat{e}_L$$

$$= e^{j\left(\frac{\theta_- + \theta_+}{2}\right)}\left[\frac{e^{j[\theta + (\theta_-/2 - \theta_+/2)]}}{\sqrt{2}}\hat{e}_R + \frac{e^{-j(\theta + \theta_-/2 - \theta_+/2)}}{\sqrt{2}}\hat{e}_L\right]$$

So compare to the 1st line in this problem solution ..

$$\vec{E} = e^{j\left(\frac{\theta_- + \theta_+}{2}\right)}\left[\frac{e^{j\phi}}{\sqrt{2}}\hat{e}_R + \frac{e^{-j\phi}}{\sqrt{2}}\hat{e}_L\right]$$

with $\phi = \theta + \dfrac{(\theta_- - \theta_+)}{2}$ instead of θ only,

i.e the plane of polarization is rotated by an angle of $(\theta_- - \theta_+)/2$.