HW6 Solution

1) $\lambda = 1.3\mu m$ $L = 20km$ $A_e = 50\mu m^2$ $\alpha_{dB} = 0.5dB/km$ $g_R = 10^{-13} m/W$

$loss_{dB} = 16dB$ $\mathrm{P_{RX}} = 0.02\mathrm{mW} = 10\log_{10}(0.02)(\mathrm{dBm}) = -16.99(\mathrm{dBm})$

(a) $(loss_{dB})_{total} = 16 + 20 \times 0.5 = 26(dB)$.

$P_{RX}(dBm) = P_{TX}(dBm) - (loss_{dB})_{total}(dB) \Rightarrow P_{TX} = -16.99 + 26 = 9.01(dBm) = 7.962(mW)$

(b) $\alpha = \alpha_{dB} / 4.343 = 0.1151(km^{-1})$, $\alpha L = 2.3 \Rightarrow$ We should use exact

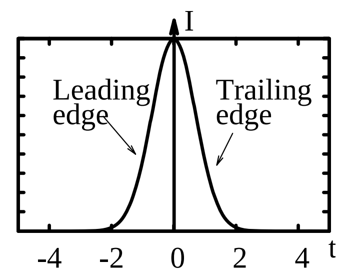
$L_e = (1 - e^{-\alpha L})/\alpha = 7.819(km)$ instead of $L_e \cong 1/\alpha = 8.688(km)$

$$P_{th} = \frac{16A_e}{g_R L_e} = \frac{16 \times 50 \times 10^{-12}}{10^{-13} \times 7.819 \times 10^3} = 1.023(W)$$

(c) $\# of\ Channels = Integer\left[\frac{P_{th}}{P_{TX}}\right] = 128$

2) FWM: f1+f2-f3
For all 3 distinctive frequencies: fa+fb-fc, fa+fc-fb, fb+fc-fa
For 2 frequencies equal: 2fa-fb, 2fa-fc, 2fb-fc, 2fc-fa, 2fc-fa, 2fc-fb
For all 3 frequencies equal: fa, fb, fc
Sum frequency generation: f1+f2+f3
For all 3 distinctive frequencies: fa+fb+fc
For 2 frequencies equal: 2fa+fb, 2fa+fc, 2fb+fa, 2fb+fc, 2fa+fa, 2fc+fb
For all 3 frequencies equal: 3fa, 3fb, 3fc.

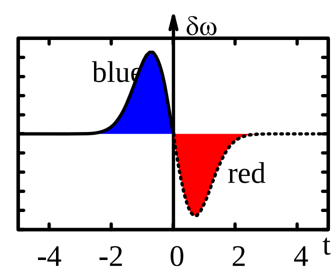
3) Another combination that can probably lead to generation of solitons is negative nonlinearity in a normal dispersive medium.



SPM causes phase change with time that results in frequency chirping (shift) , i.e.

$$\delta\omega = -\frac{d\Delta\phi}{dt} = -\frac{d\frac{2\pi}{\lambda}\Delta n L}{dt} = -\frac{2\pi}{\lambda} n_2 L \frac{dI}{dt} \ (\text{note}: \Delta n = n_2 I)$$

where I is intensity, L is the length of fiber, n_2 nonlinear refractive index coefficient. Nagative or self-defocusing medium means $n_2 < 0$, i.e. $\delta\omega = \frac{2\pi}{\lambda} |n_2| L \frac{dI}{dt}$.



From the diagram, you see the leading edge of the pulse is blue-shifted and the trailing edge of the pulse is red-shifted.
For normal dispersion, $D_{intra} < 0$, i.e. time delay increases as wavelength decreases or high frequency (blue) is slower (on the trailing edge) which is just opposite of the δω introduced by SPM. Hence the normal dispersion cancel the frequency chirping induced by SPM.

4) Soliton formation at λ=1.58μm and D_{intra}=30 ps/km-nm. Fiber has

$n_2 = 3.19\times10^{-16}(\text{cm}^2/\text{W})$ and $a = 5\mu\text{m}$

a) $$\gamma = \frac{2\pi}{\lambda A_e} n_2 = \frac{2\pi}{1.58\times10^{-6}\times\pi\left(5\times10^{-6}\right)^2}\times3.19\times10^{-16}\times10^{-4} = 1.615\times10^{-3}(\text{rad}/\text{W}-\text{m})$$

$$= 1.615(\text{rad}/\text{W}-\text{km})$$

b) Find soliton pulse width at power of 10mW.

$$D_{intra} = -K''\frac{2\pi c}{\lambda^2} \Rightarrow K'' = -D_{intra}\frac{\lambda^2}{2\pi c}$$

$$= -30\times10^{-12}\times10^{-3}\times10^{9}\times\frac{\left(1.58\times10^{-6}\right)^2}{2\pi\times3\times10^{8}} = -3.973\times10^{-26}\,\text{s}^2/\text{m} = -39.73\text{ps}^2/\text{km}$$

$$A_o = \sqrt{P_o} = 0.1(\sqrt{\text{W}})$$

$$T_o = \sqrt{\frac{|K''|}{\gamma P_o}} = \sqrt{\frac{39.73}{1.615\times0.01}} = 49.75(\text{ps})$$

c) To achieve short pulse width, we will need to increase power. For example, a pulse width of 4.975ps will require a power of 1W.

5) $\lambda = 1.6\mu m$, $L = 40km$ $A_e = 45\mu m^2$, $n_2 = 3.2\times10^{-20}(\text{m}^2/\text{W})$,

$$\alpha_{dB} = 0.3dB/km \Rightarrow \alpha = 0.3/4.343 = 6.908\times10^{-2}(km^{-1})$$

$\gamma = \frac{2\pi}{\lambda A_e} n_2 = \frac{2\pi}{1.6\times10^{-6}\times\left(45\times10^{-12}\right)}\times3.2\times10^{-20} = 2.793\times10^{-3}(\text{rad}/\text{W}-\text{m})$. αL = 2.8. We should use the exact expression for L_e: $L_e = (1-e^{-\alpha L})/\alpha = 13.56(km)$

a) For SPM causing a phase shift of π,

$$\Delta\varphi_1 = \gamma L_e P_1 = \pi \Rightarrow P_1 = \pi/(\gamma L_e) = \pi/(2.793\times10^{-3}\times13.56\times10^{3}) = 82.95(\text{mW})$$

b) $P_2 = 80\text{mW}$ and $P_3 = 40\text{mW}$

For XPM causing a phase shift of π,

$$\Delta\varphi_1 = \gamma L_e 2(P_2+P_3) = \pi \Rightarrow L_e = \pi/(\gamma 2(P_2+P_3)) = \pi/(2.793\times10^{-3}\times2\times0.12) = 4.687(\text{km})$$

$$L_e = (1-e^{-\alpha L})/\alpha \Rightarrow L = -\ln(1-\alpha L_e)/\alpha = 5.663(\text{km})$$

Extra-Credit

$$\frac{dE}{dz} = -jk_0 n_2 |E|^2 E \quad \text{\&} \quad E = E_{10} e^{j\omega_1 t} + E_{20} e^{j\omega_2 t}$$

① — $|E|^2 E = E E^* E = (E_{10} e^{j\omega_1 t} + E_{20} e^{j\omega_2 t})(E_{10}^* e^{-j\omega_1 t} + E_{20}^* e^{-j\omega_2 t})$

$(E_{10} e^{j\omega_1 t} + E_{20} e^{j\omega_2 t})$

$= \left(|E_{10}|^2 + |E_{20}|^2 + E_{10} E_{20}^* e^{j\omega_1 t - j\omega_2 t} + E_{20} E_{10}^* e^{j(\omega_2 - \omega_1)t}\right)$

$(E_{10} e^{j\omega_1 t} + E_{20} e^{j\omega_2 t})$

$= \left[(|E_{10}|^2 + |E_{20}|^2) E_{10} + E_{10} |E_{20}|^2\right] e^{j\omega_1 t} + \left[|E_{10}|^2 E_{20} + (|E_{10}|^2 + |E_{20}|^2) E_{20}\right] e^{j\omega_2 t} + E_{10}^2 E_{20}^* e^{j(2\omega_1 - \omega_2)t} + E_{20}^2 E_{10}^* e^{j(2\omega_2 - \omega_1)t}$

neglected →

② — $dE/dz = e^{j\omega_1 t} dE_{10}/dz + e^{j\omega_2 t} dE_{20}/dz$

② = $-jk_0 n_2 \times$ ① $\Rightarrow \frac{dE_{10}}{dz} = -jk_0 n_2 (|E_{10}|^2 + 2|E_{20}|^2) E_{10}$ //

SPM, XPM

$\frac{dE_{20}}{dz} = -jk_0 n_2 (|E_{20}|^2 + 2|E_{10}|^2) E_{20}$ //