HW 7 Solution

1) $R_1 = R_2 = 0.9$, $\lambda_o = 1.5\mu m$, $d = 1.5mm$, $n = 1.46$

a) $\Delta f_{FSR} = \dfrac{c}{2nd} = \dfrac{3 \times 10^8}{2 \times 1.46 \times 1.5 \times 10^{-3}} = 6.849 \times 10^{10}$ (Hz)

b) $\mathscr{F} = \dfrac{\pi\sqrt{R}}{1-R} = \dfrac{\pi\sqrt{0.9}}{1-0.9} = 29.81$

b) $\mathscr{F} = \dfrac{\Delta f_{FSR}}{\Delta \nu} \Rightarrow \Delta\nu = \dfrac{6.849 \times 10^{10}}{29.8} = 2.3 \times 10^9$ (nm)

c) $\dfrac{\Delta\lambda}{\lambda_o} = \dfrac{\Delta\nu}{\nu} \Rightarrow \Delta\nu = \dfrac{\lambda_o^2}{c}\Delta\nu = 2.3 \times 10^9 \dfrac{\left(1.5 \times 10^{-6}\right)^2}{3 \times 10^8} = 0.01724\,(\text{nm})$

d) $d = \dfrac{m\lambda_o}{2}n \Rightarrow m = round\left(\dfrac{2nd}{\lambda_o}\right) = 2920$

e) Since m is an exact integer, λ_o should be the intented wavelength, i.e. exact
$\lambda_o = \dfrac{2nd}{m} = 1.5(\mu m)$

2) 16 Channels, 100 GHz spacing, n = 1.5, $\Delta\nu = 2GHz$

i) $\Delta f_{FSR} \approx 100 \times 16 = 1600$ (GHz)

$\Delta f_{FSR} = \dfrac{c}{2nd} \Rightarrow d = \dfrac{c}{2n\Delta f_{FSR}} = \dfrac{3 \times 10^8}{2 \times 1.5 \times 1600 \times 10^9} = 0.0625$ (mm)

ii) $\mathscr{F} = \dfrac{\Delta f_{FSR}}{\Delta\nu} \approx \dfrac{1600 \times 10^9}{2 \times 10^9} = 800$

iii) $\mathscr{F} = \dfrac{\pi\sqrt{R}}{1-R} = \dfrac{\pi x}{1-x^2}$ where $x = \sqrt{R} \Rightarrow x^2 + \dfrac{\pi}{\mathscr{F}}x - 1 = 0$

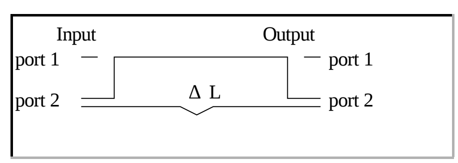
$$x = \frac{-\frac{\pi}{\mathscr{F}} \pm \sqrt{\left(\frac{\pi}{\mathscr{F}}\right)^2 + 4}}{2} \approx -\frac{\pi}{2\mathscr{F}} \pm \left(1 + 0.5\left(\frac{\pi}{2\mathscr{F}}\right)^2\right)$$

Choose solution of x that is less than 1, $x = -\dfrac{\pi}{2\mathscr{F}} + \left(1 + 0.5\left(\dfrac{\pi}{2\mathscr{F}}\right)^2\right) = 1 - 1.962 \times 10^{-3}$

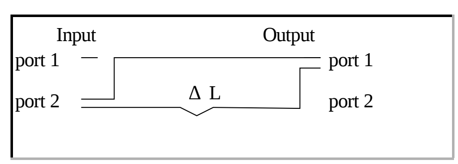
$R = x^2 \approx 1 - 2 \times 1.962 \times 10^{-3} = 0.9961$

3) a) Consider Fig. 3.21,

At output port 2:
Upper path: $k_2 L + 2 \times \dfrac{\pi}{2}$
Lower path: $k_2(L + \Delta L)$
Condition for constructive interference:
$k_2(L + \Delta L) - \left(k_2 L + 2 \times \dfrac{\pi}{2}\right) = 2\pi m$
$k_2 \Delta L - \pi = 2\pi m$
$\Rightarrow \dfrac{2\pi}{c} f_2 n \Delta L = (2m+1)\pi$



At output port 1:
Upper path: $k_1 L + \dfrac{\pi}{2}$
Lower path: $k_1(L + \Delta L) + \dfrac{\pi}{2}$
Condition for constructive interference:



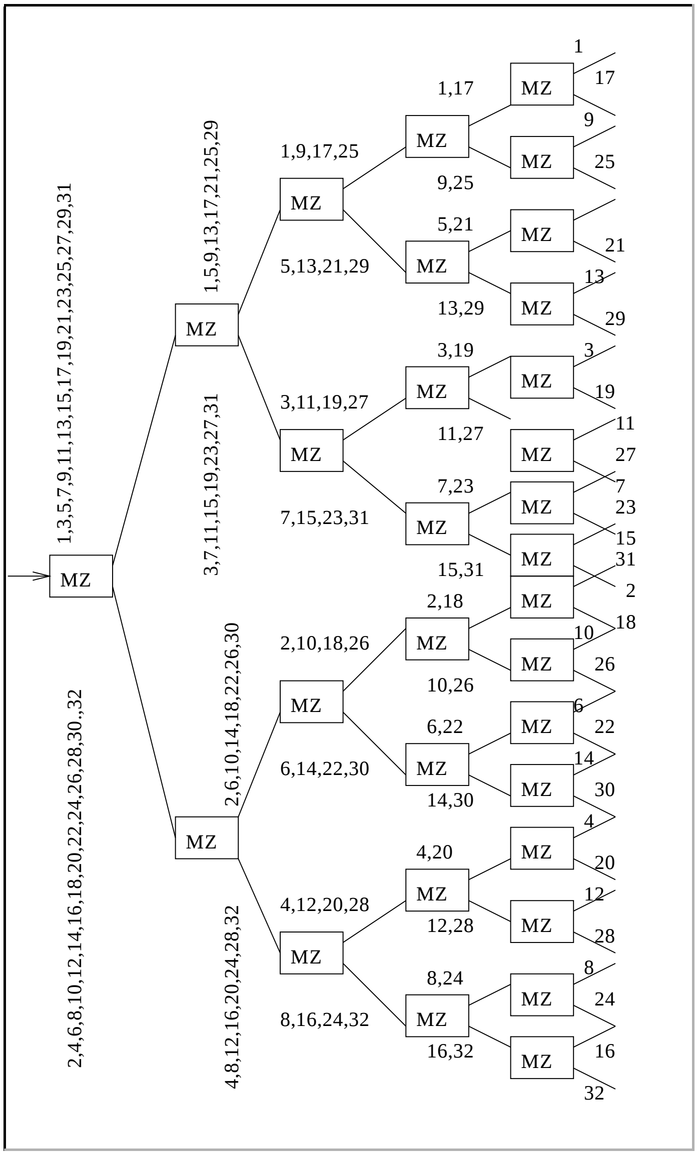
$k_1(L+\Delta L)+\frac{\pi}{2}-\left(k_1 L+\frac{\pi}{2}\right)=2\pi m$
$k_1\Delta L=2\pi m$
$\Rightarrow \frac{2\pi}{c}f_1 n\Delta L=2m\pi$

b) $\Delta f=25$ GHz, n = 1.45, $\Delta L=$?
$\Delta f=f_2-f_1=\frac{c}{2n\Delta L}\Rightarrow \Delta L=\frac{c}{2n\Delta f}=\frac{3\times 10^8}{2\times 1.45\times 2.5\times 10^{10}}=4.138\ \text{(mm)}$

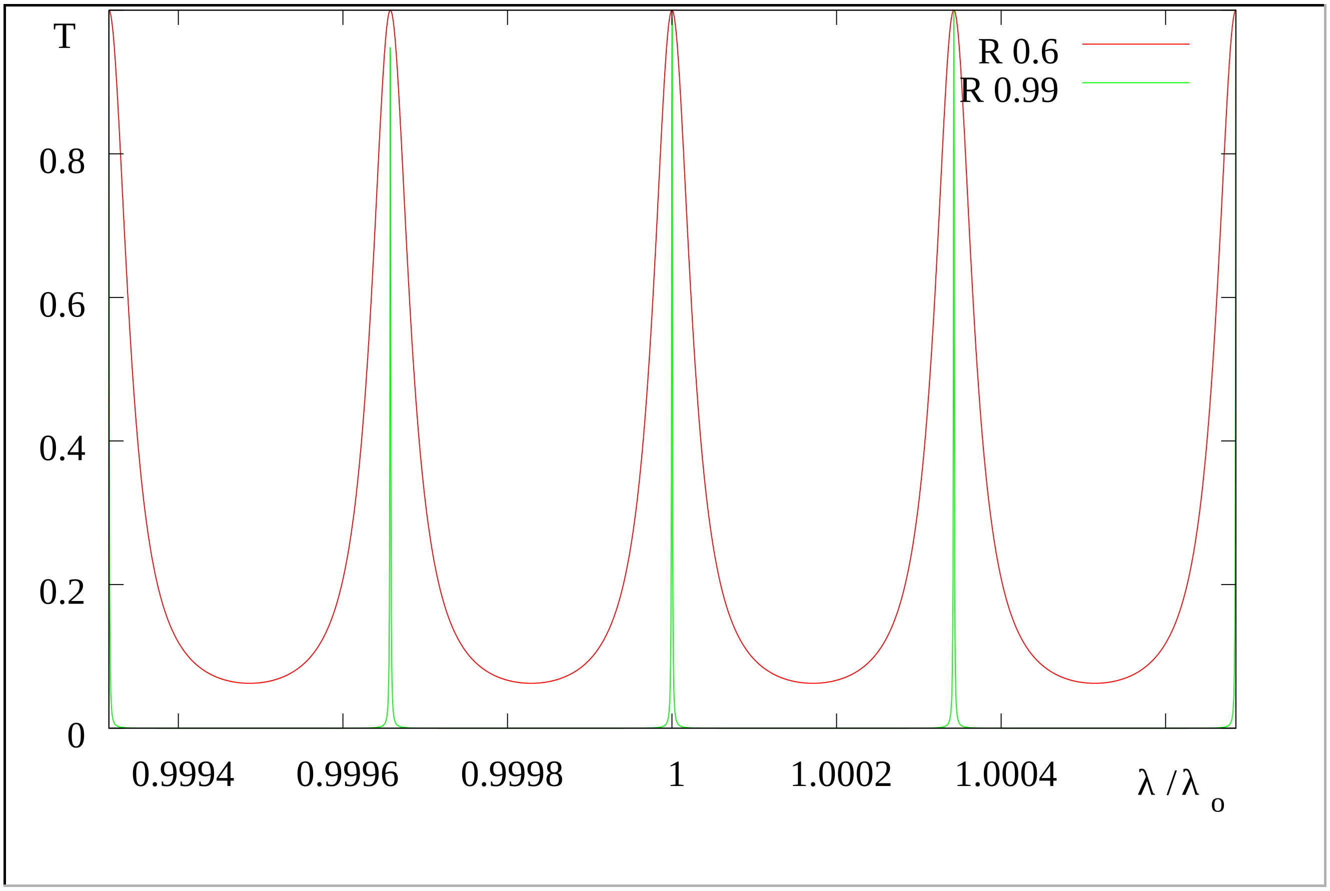
4) $D_{\text{intra}}=25p\frac{s}{km-nm}$, $E_g=0.36+2.012x+0.698x^2$, $E(ev)=\frac{1.24}{\lambda(\mu m)}$
a) x =0.4, $E_g=0.36+2.012\times 0.4+0.698\times 0.4^2=1.276$(ev)
b) At 273K, $E=E_g+kT=1.276+26\times 10^{-3}\times\frac{273}{300}=1.300\,\text{(ev)}$, $\lambda=\frac{1.24}{1.300}=0.9537(\mu m)$
At 323K, $E=E_g+kT=1.276+26\times 10^{-3}\times\frac{323}{300}=1.304\,\text{(ev)}$, $\lambda=\frac{1.24}{1.304}=0.9509(\mu m)$
c) $\Delta E=3.3kT$ and at 273K, $\Delta E=3.3\times 26\times 10^{-3}\times\frac{273}{300}=0.07808\,\text{(ev)}$
$\frac{\Delta E}{E}=\frac{\Delta\lambda}{\lambda}\Rightarrow\Delta\lambda=\frac{\Delta E}{E}\lambda=\frac{\Delta E}{E}\frac{1.24}{E}=\frac{0.07808\times 1.24}{1.3^2}=0.0574(\mu m)$
At 323K, $\Delta E=3.3\times 26\times 10^{-3}\times\frac{323}{300}=0.092378\,\text{(ev)}$
$\frac{\Delta E}{E}=\frac{\Delta\lambda}{\lambda}\Rightarrow\Delta\lambda=\frac{\Delta E}{E}\lambda=\frac{\Delta E}{E}\frac{1.24}{E}=\frac{0.092378\times 1.24}{1.304^2}=0.0677(\mu m)$
d) $R_b=1G\frac{b}{s}$, $\Delta\tau=D_{\text{intra}}\Delta\lambda L$, $R_b=\frac{1}{4\Delta\tau}\Rightarrow L=\frac{1}{4R_b D_{\text{intra}}\Delta\lambda}$
At 273K, $L=\frac{1}{4\times 10^9\times 25\times 57.38\times 10^{-12}}=0.174\,\text{(km)}$
At 323K, $L=\frac{1}{4\times 10^9\times 25\times 67.7\times 10^{-12}}=0.1477\,\text{(km)}$

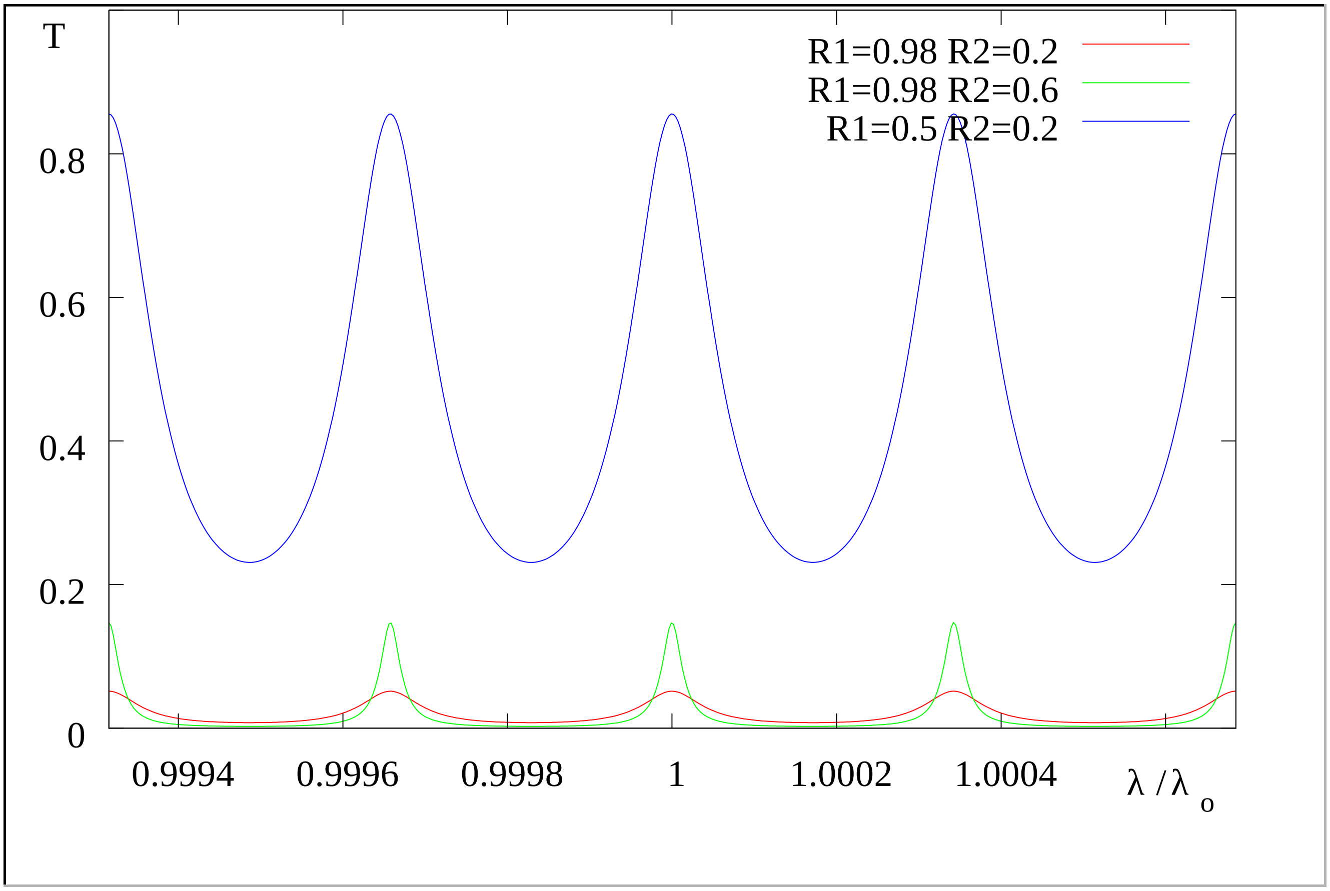
5) a) $32=2^m\Rightarrow m=5$, i.e. 5 layers or stages and number of MZ filters = 1 +2 +4 + 8 +16 = 31.
b)

c) Stage 1: $\Delta f=30$ GHz
$\Delta L=\frac{c}{2n\Delta f}=\frac{3\times 10^8}{2\times 1.5\times 3\times 10^{10}}=3.33$ (mm)
Stage 2: $\Delta f_2=60$ GHz
$\Delta L_2=\frac{\Delta L_1}{2}=1.67\text{ (mm)}$
Stage 3: $\Delta f_3=120$ GHz
$\Delta L_3=\frac{\Delta L_2}{2}=0.833\,\text{(mm)}$
Stage 4: $\Delta f_4=240$ GHz
$\Delta L_4=\frac{\Delta L_3}{2}=0.4167\,\text{(mm)}$
Stage 5: $\Delta f_5=480$ GHz
$\Delta L_5=\frac{\Delta L_4}{2}=0.2089\,\text{(mm)}$



6) To ensure the plot having 3-4 peaks, we choose the range for $m = 2920 \pm 2$ and normalize the wavelength axis by the exact wavelength, i.e. $\frac{\lambda}{\lambda_o}$ so that we get nice numbers on horizontal axis. I included scripts for "gnuplot" to show the plotting. This program can generate nice plot quickly.


T
R 0.6
R 0.99
0.8
0.6
0.4
0.2
0
0.9994
0.9996
0.9998
1
1.0002
1.0004
λ / λ o


T
R1=0.98 R2=0.2
R1=0.98 R2=0.6
R1=0.5 R2=0.2
0.8
0.6
0.4
0.2
0
0.9994
0.9996
0.9998
1
1.0002
1.0004
λ / λ o