

Figure 2.5-5 The interference of a plane wave and a spherical wave creates a pattern of concentric rings (illustrated at the plane $z = d$).

EXERCISE 2.5-2

Interference of Two Spherical Waves. Two spherical waves of equal intensity I_0 , originating at the points $(-a, 0, 0)$ and $(a, 0, 0)$, interfere in the plane $z = d$ as illustrated in Fig. 2.5-6. This double-pinhole system is similar to that used by Thomas Young in his celebrated double-slit experiment in which he demonstrated interference. Use the paraboloidal approximation for the spherical waves to show that the intensity at the plane $z = d$ is

$$I(x, y, d) \approx 2I_0 \left(1 + \cos \frac{2\pi x\theta}{\lambda} \right), \quad (2.5-8)$$

where the angle subtended by the centers of the two waves at the observation plane is $\theta \approx 2a/d$. The intensity pattern is periodic with period λ/θ .

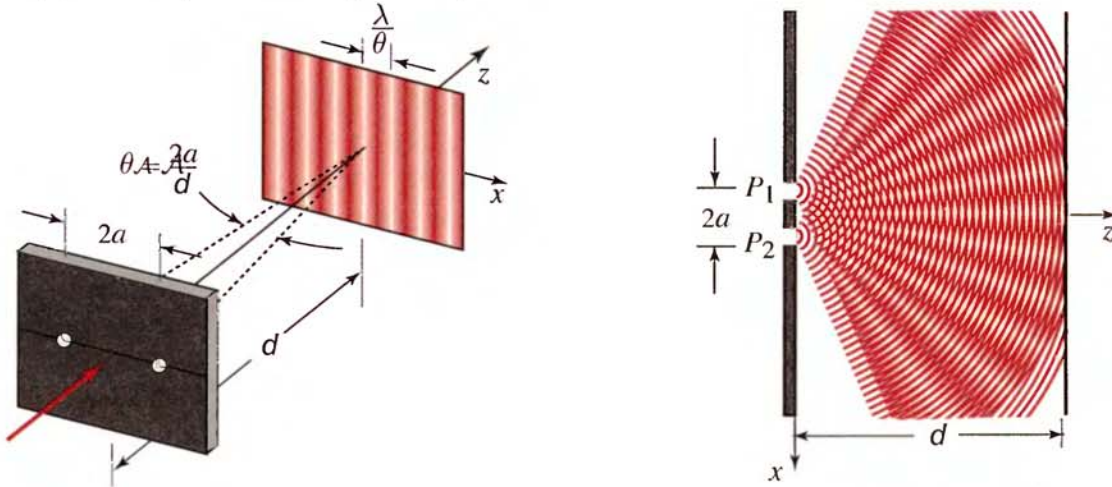


Figure 2.5-6 Interference of two spherical waves of equal intensities originating at the points P_1 and P_2 . The two waves can be obtained by permitting a plane wave to impinge on two pinholes in a screen. The light intensity at an observation plane a large distance d from the pinholes takes the form of a sinusoidal interference pattern, with period $\approx \lambda/\theta$, along the direction of the line connecting the pinholes.

B. Multiple-Wave Interference

The superposition of M monochromatic waves of the same frequency, with complex amplitudes U_1, U_2, \dots, U_M , gives rise to a wave whose frequency remains the same and whose complex amplitude is given by $U = U_1 + U_2 + \dots + U_M$. Knowledge of the intensities of the individual waves, I_1, I_2, \dots, I_M , is not sufficient to determine the total intensity $I = |U|^2$ since the relative phases must also be known. The role played by the phase is dramatically illustrated in the following examples.

Interference of M Waves with Equal Amplitudes and Equal Phase Differences

We first examine the interference of M waves with complex amplitudes

$$U_m = \sqrt{I_0} \exp[j(m-1)\varphi], \quad m = 1, 2, \dots, M. \quad (2.5-9)$$

The waves have equal intensities I_0 , and phase difference φ between successive waves, as illustrated in Fig. 2.5-7(a). To derive an expression for the intensity of the superposition, it is convenient to introduce the quantity $h = \exp(j\varphi)$ whereupon $U_m = \sqrt{I_0} h^{m-1}$. The complex amplitude of the superposed wave is then

$$\begin{aligned} U &= \sqrt{I_0} (1 + h + h^2 + \dots + h^{M-1}) = \sqrt{I_0} \frac{1 - h^M}{1 - h} \\ &= \sqrt{I_0} \frac{1 - \exp(jM\varphi)}{1 - \exp(j\varphi)}, \end{aligned} \quad (2.5-10)$$

which has the corresponding intensity

$$I = |U|^2 = I_0 \left| \frac{\exp(-jM\varphi/2) - \exp(jM\varphi/2)}{\exp(-j\varphi/2) - \exp(j\varphi/2)} \right|^2, \quad (2.5-11)$$

whence

$$I = I_0 \frac{\sin^2(M\varphi/2)}{\sin^2(\varphi/2)}. \quad (2.5-12)$$

Interference of M Waves

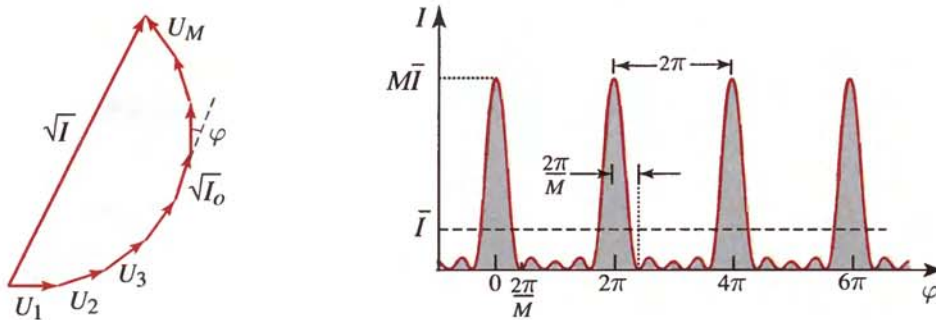


Figure 2.5-7 (a) The sum of M phasors of equal magnitudes and equal phase differences. (b) The intensity I as a function of φ . The peak intensity occurs when all the phasors are aligned; it is then M times greater than the mean intensity $\bar{I} = MI_0$. In this example $M = 5$.

The intensity I is evidently strongly dependent on the phase difference φ , as illustrated in Fig. 2.5-7(b) for $M = 5$. When $\varphi = 2\pi q$, where q is an integer, all the phasors are aligned so that the amplitude of the total wave is M times that of an individual component, and the intensity reaches its peak value of $M^2 I_0$. The mean intensity averaged over a uniform distribution of φ is $\bar{I} = (1/2\pi) \int_0^{2\pi} I d\varphi = MI_0$, which is the same as the result obtained in the absence of interference. The peak intensity is therefore M times greater than the mean intensity. The sensitivity of the intensity to the

phase is therefore dramatic for large M . At its peak value, the intensity is magnified by a factor M over the mean but it decreases sharply as the phase difference φ deviates slightly from $2\pi q$. In particular, when $\varphi = 2\pi/M$ the intensity becomes zero. It is instructive to compare Fig. 2.5-7(b) for $M = 5$ with Fig. 2.5-2 for $M = 2$.

EXERCISE 2.5-3

Bragg Reflection. Consider light reflected at an angle θ from M parallel reflecting planes separated by a distance Λ , as shown in Fig. 2.5-8. Assume that only a small fraction of the light is reflected from each plane, so that the amplitudes of the M reflected waves are approximately equal. Show that the reflected waves have a phase difference $\varphi = k(2\Lambda \sin \theta)$ and that the angle θ at which the intensity of the total reflected light is maximum satisfies

$$\sin \theta = \frac{\lambda}{2\Lambda}. \quad (2.5-13)$$

Bragg Angle

This equation defines the **Bragg angle** θ . Such reflections are encountered when light is reflected from a multilayer structure (see Sec. 7.1) or when X-ray waves are reflected from atomic planes in crystalline structures. It also occurs when light is reflected from a periodic structure created by an acoustic wave (see Chapter 19). An exact treatment of Bragg reflection is provided in Sec. 7.1C.

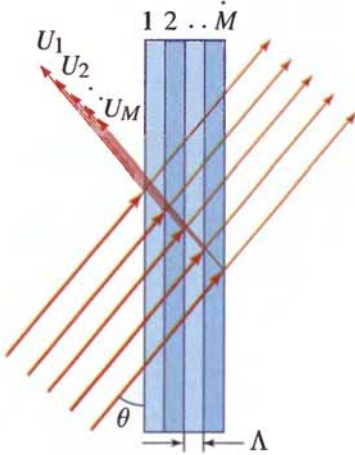


Figure 2.5-8 Reflection of a plane wave from M parallel planes separated from each other by a distance Λ . The reflected waves interfere constructively and yield maximum intensity when the angle θ is the Bragg angle. Note that θ is defined with respect to the parallel planes.

Interference of an Infinite Number of Waves of Progressively Smaller Amplitudes and Equal Phase Differences

We now examine the superposition of an infinite number of waves with equal phase differences and with amplitudes that decrease at a geometric rate:

$$U_1 = \sqrt{I_0}, \quad U_2 = hU_1, \quad U_3 = hU_2 = h^2U_1, \quad \dots, \quad (2.5-14)$$

where $h = |h|e^{j\varphi}$, $|h| < 1$, and I_0 is the intensity of the initial wave. The amplitude of the m th wave is smaller than that of the $(m - 1)$ st wave by the factor $|h|$ and the phase differs by φ . The phasor diagram is shown in Fig. 2.5-9(a).

The superposition wave has a complex amplitude

$$\begin{aligned} U &= U_1 + U_2 + U_3 + \dots \\ &= \sqrt{I_0} (1 + h + h^2 + \dots) \end{aligned}$$