Interference of an Infinite Number of Waves of Progressively Smaller Amplitudes and Equal Phase Differences

We now examine the superposition of an infinite number of waves with equal phase differences and with amplitudes that decrease at a geometric rate:

$$U_1 = \sqrt{I_0}\,, \quad U_2 = hU_1, \quad U_3 = hU_2 = h^2 U_1, \quad \ldots, \tag{2.5-14}$$

where $h = |h|e^{j\varphi}$, $|h| < 1$, and I_0 is the intensity of the initial wave. The amplitude of the mth wave is smaller than that of the $(m-1)$st wave by the factor $|h|$ and the phase differs by φ. The phasor diagram is shown in Fig. 2.5-9(a).

The superposition wave has a complex amplitude

$$\begin{aligned} U &= U_1 + U_2 + U_3 + \cdots \\ &= \sqrt{I_0}\,(1 + h + h^2 + \cdots) \end{aligned}$$

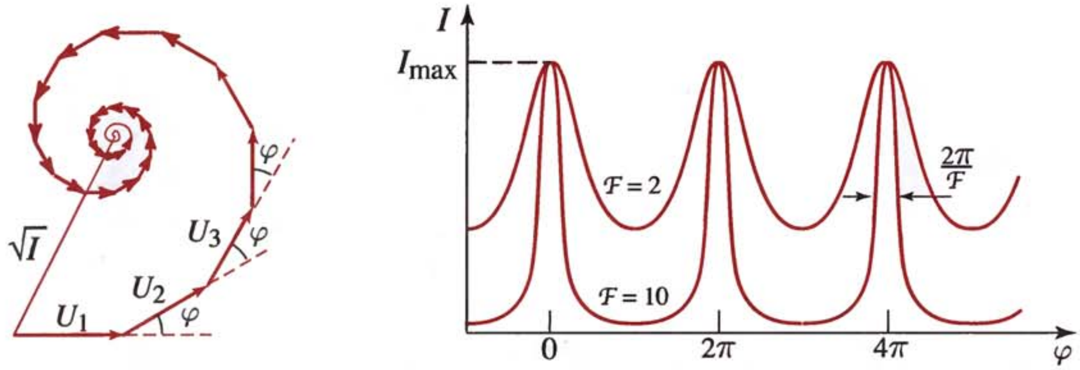

Figure 2.5-9 (*a*) The sum of an infinite number of phasors whose magnitudes are successively reduced at a geometric rate and whose phase differences φ are equal. (*b*) Dependence of the intensity I on the phase difference φ for two values of $\mathcal{F}$. Peak values occur at $\varphi = 2\pi q$. The full width at half maximum of each peak is approximately $2\pi/\mathcal{F}$ when $\mathcal{F} \gg 1$. The sharpness of the peaks increases with increasing $\mathcal{F}$.

$$= \frac{\sqrt{I_0}}{1-h} = \frac{\sqrt{I_0}}{1-|h|e^{j\varphi}}. \tag{2.5-15}$$

The total intensity is then

$$I = |U|^2 = \frac{I_0}{|1-|h|e^{j\varphi}|^2} = \frac{I_0}{(1-|h|\cos\varphi)^2 + |h|^2\sin^2\varphi}, \tag{2.5-16}$$

from which

$$I = \frac{I_0}{(1-|h|)^2 + 4|h|\,\sin^2(\varphi/2)}. \tag{2.5-17}$$

It is convenient to write this equation in the form

$$\boxed{I = \frac{I_{max}}{1+(2\mathcal{F}/\pi)^2\sin^2(\varphi/2)},} \qquad I_{max} = \frac{I_0}{(1-|h|)^2}, \tag{2.5-18}$$

Intensity of an Infinite Number of Waves

where the quantity

$$\boxed{\mathcal{F} = \frac{\pi\sqrt{|h|}}{1-|h|}} \tag{2.5-19}$$

Finesse

is a parameter known as the **finesse**.

The intensity I is a periodic function of φ with period 2π, as illustrated in Fig. 2.5-9(*b*). It reaches its maximum value I_{max} when $\varphi = 2\pi q$, where q is an integer. This occurs when the phasors align to form a straight line. (This result is not unlike that displayed in Fig. 2.5-7(*b*) for the interference of M waves of equal amplitudes and equal phase differences.) When the finesse $\mathcal{F}$ is large (i.e., the factor $|h|$ is close to 1), I becomes a sharply peaked function of φ. Consider values of φ near the $\varphi = 0$ peak, as a representative example. For $|\varphi| \ll 1$, $\sin(\varphi/2) \approx \varphi/2$ whereupon (2.5-18) can be written as

$$I \approx \frac{I_{\max}}{1 + (\mathcal{F}/\pi)^2 \varphi^2}. \qquad (2.5\text{-}20)$$

The intensity I then decreases to half its peak value when $\varphi = \pi/\mathcal{F}$, so that the full width at half maximum (FWHM) of the peak becomes

$$\boxed{\Delta\varphi \approx \frac{2\pi}{\mathcal{F}}.} \qquad (2.5\text{-}21)$$

Width of Interference Pattern

In the regime $\mathcal{F} \gg 1$, we then have $\Delta\varphi \ll 2\pi$ and the assumption that $\varphi \ll 1$ is applicable. The finesse $\mathcal{F}$ is the ratio of the period 2π to the FWHM of the peaks in the interference pattern. It is therefore a measure of the sharpness of the interference function, i.e., the sensitivity of the intensity to deviations of φ from the values $2\pi q$ corresponding to the peaks.

A useful device based on this principle is the Fabry–Perot interferometer. It consists of two parallel mirrors within which light undergoes multiple reflections. In the course of each round trip, the light suffers a fixed amplitude reduction $|h| = |\mathrm{r}|$, arising from losses at the mirrors, and a phase shift $\varphi = k2d = 4\pi\nu d/c = 2\pi\nu/\,(c/2d)$ associated with the propagation, where d is the mirror separation. The total light intensity depends on the phase shift φ in accordance with (2.5-18), attaining maxima when $\varphi/2$ is an integer multiple of π. The proportionality of the phase shift φ to the optical frequency ν shows that the intensity transmission of the Fabry–Perot device will exhibit peaks separated in frequency by $c/2d$. The width of these peaks will be $(c/2d)/\mathcal{F}$, where the finesse $\mathcal{F}$ is governed by the loss via (2.5-19). The Fabry–Perot interferometer, which also serves as a spectrum analyzer, is considered further in Sec. 7.1B. It is commonly used as a resonator for lasers, as discussed in Secs. 10.1 and 15.1A.

2.6 POLYCHROMATIC AND PULSED LIGHT

Since the wavefunction of monochromatic light is a harmonic function of time extending over all time (from $-\infty$ to ∞), it is an idealization that cannot be met in reality. This section is devoted to waves of arbitrary time dependence, including optical pulses of finite time duration. Such waves are polychromatic rather than monochromatic. A more detailed introduction to the optics of pulsed light is provided in Chapter 22.

A. Temporal and Spectral Description

Although a polychromatic wave is described by a wavefunction $u(\mathbf{r}, t)$ with nonharmonic time dependence, it may be expanded as a superposition of harmonic functions, each of which represents a monochromatic wave. Since we already know how monochromatic waves propagate in free space and through various optical components, we can determine the effect of optical systems on polychromatic light by using the principle of superposition.

Fourier methods permit the expansion of an arbitrary function of time $u(t)$, representing the wavefunction $u(\mathbf{r}, t)$ at a fixed position $\mathbf{r}$, as a superposition integral of