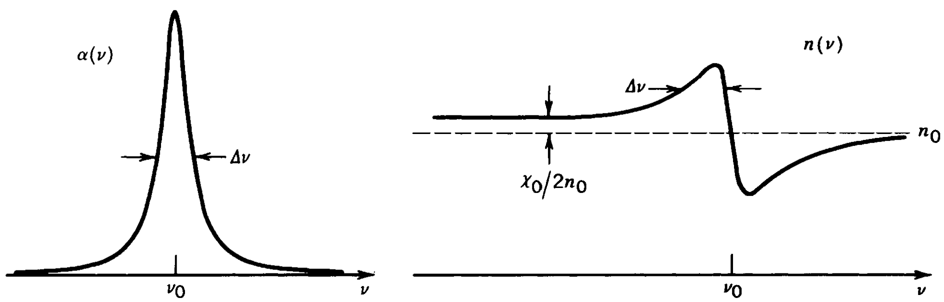

Figure 5.5-6 The absorption coefficient $\alpha(\nu)$ and refractive index $n(\nu)$ of a dielectric medium of refractive index n_0 with dilute atoms of resonance frequency ν_0.

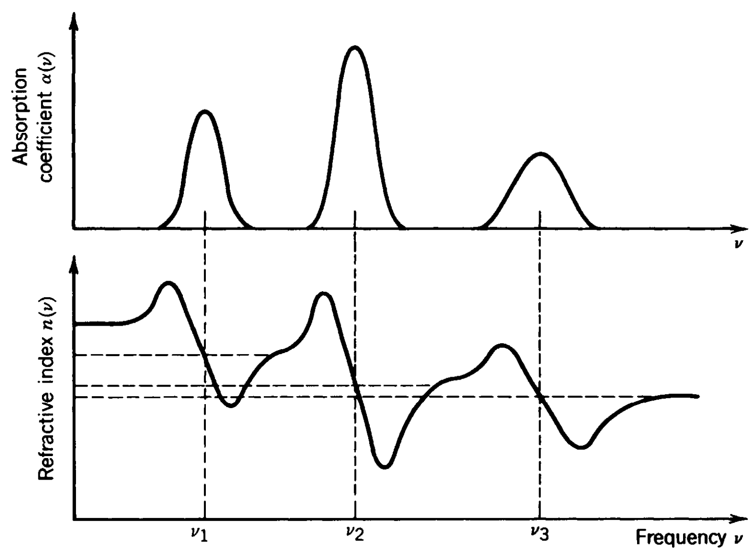

Figure 5.5-7 Absorption coefficient $\alpha(\nu)$ and refractive index $n(\nu)$ of a medium with three resonances.

mately nondispersive and nonabsorptive. Each resonance contributes a constant value to the refractive index at all frequencies smaller than its resonance frequency.

Other complex processes can also contribute to the absorption coefficient and the refractive index, so that different patterns of wavelength dependence can be exhibited. Figure 5.5-8 shows an example of the wavelength dependence of the absorption coefficient and refractive index for a dielectric material that is essentially transparent to light at visible wavelengths. In the visible band, the refractive index varies slightly because of proximity to ultraviolet absorption. In this band the refractive index is a decreasing function of wavelength. The rate of decrease is greater at shorter wavelengths, so that the material is more dispersive at short wavelengths.

5.6 PULSE PROPAGATION IN DISPERSIVE MEDIA

The study of pulse propagation in dispersive media is important in many applications, including the transmission of optical pulses through the glass fibers used in optical communication systems (as will become clear in Chaps. 8 and 22). The dispersive

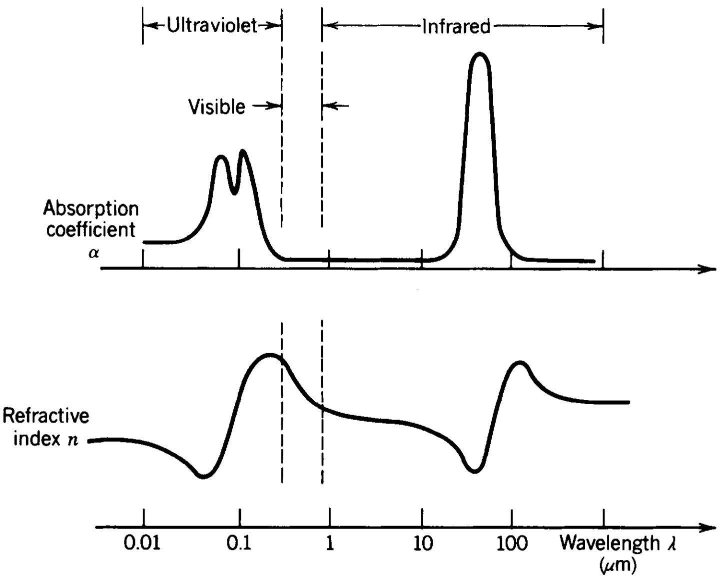

Figure 5.5-8 Typical wavelength dependence of the absorption coefficient and the refractive index of a dielectric material exhibiting resonance absorptions in the ultraviolet and infrared bands and low absorption in the visible band.

medium is characterized by a frequency-dependent refractive index, absorption coefficient, and phase velocity, so that monochromatic waves of different frequencies travel in the medium at different velocities and undergo different attenuations. Since a pulse of light is the sum of many monochromatic waves, each of which is modified differently, the pulse is delayed and broadened (dispersed in time) and its shape is altered. In this section we determine the velocity of a pulse, the rate at which it spreads in time, and the changes in its shape, as it travels through a dispersive medium.

Consider a pulsed plane wave traveling in the z direction in a linear, homogeneous, and isotropic medium with absorption coefficient $\alpha(\nu)$, refractive index $n(\nu)$, and propagation constant $\beta(\nu) = 2\pi\nu n(\nu)/c_o$. The complex wavefunction is

$$U(z,t) = \mathscr{A}(z,t)\exp[j(2\pi\nu_0 t - \beta_0 z)], \tag{5.6-1}$$

where ν_0 is the central frequency, $\beta_0 = \beta(\nu_0)$ is the central wavenumber, and $\mathscr{A}(z,t)$ is the complex envelope of the pulse, assumed to be slowly varying in comparison with the central frequency ν_0 (Fig. 5.6-1). The complex envelope $\mathscr{A}(0,t)$ in the plane $z = 0$ is assumed to be a known function, and we wish to determine $\mathscr{A}(z,t)$ at a distance z in the medium.

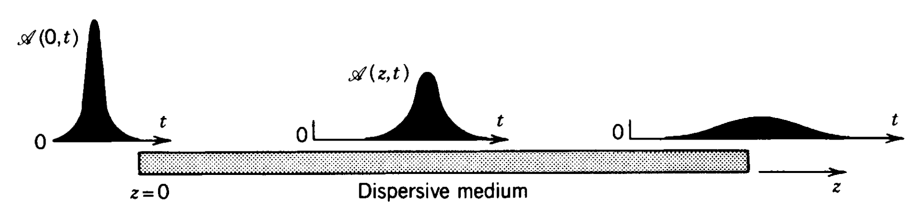

Figure 5.6-1 Broadening of the complex envelope of a pulse as a result of propagation in a dispersive medium.

Linear-System Description

The incident pulse $\mathscr{A}(0,t)$ and the transmitted pulse $\mathscr{A}(z,t)$ may be regarded as the input and output of a linear system using the techniques described in Appendix B, Sec. B.1. We aim at developing a procedure for determining $\mathscr{A}(z,t)$ from $\mathscr{A}(0,t)$.

Suppose first that the complex envelope $\mathscr{A}(0,t)$ is itself a harmonic function $\mathscr{A}(0,t) = A(0,f)\exp(j2\pi ft)$ with frequency f, so that the wave is monochromatic with frequency $\nu = f + \nu_0$. The complex wavefunction then varies with z in accordance with $U(z,t) = U(0,t)\exp[-\frac{1}{2}\alpha(f+\nu_0)z - j\beta(f+\nu_0)z]$. Using (5.6-1), $A(z,f) = A(0,f)\exp\{-\frac{1}{2}\alpha(f+\nu_0)z - j[\beta(f+\nu_0) - \beta(\nu_0)]z\}$, from which

$$A(z,f) = A(0,f)\mathscr{H}(f), \tag{5.6-2}$$

where

$$\boxed{\mathscr{H}(f) = \exp\{-\tfrac{1}{2}\alpha(f+\nu_0)z - j[\beta(f+\nu_0) - \beta(\nu_0)]z\}.} \tag{5.6-3}$$

Transfer Function

The factor $\mathscr{H}(f)$ is therefore the transfer function of the linear system whose input and output are the time functions $\mathscr{A}(0,t)$ and $\mathscr{A}(z,t)$ (see Appendix B, Sec. B.1).

We now describe a systematic procedure for determining the output $\mathscr{A}(z,t)$ from the input $\mathscr{A}(0,t)$ for an arbitrary dispersive medium. The complex envelope $\mathscr{A}(z,t)$ of an arbitrary pulse can always be decomposed as a superposition of harmonic functions by using the Fourier-transform relations,

$$\mathscr{A}(z,t) = \int_{-\infty}^{\infty} A(z,f)\exp(j2\pi ft)\,df \tag{5.6-4a}$$

$$A(z,f) = \int_{-\infty}^{\infty} \mathscr{A}(z,t)\exp(-j2\pi ft)\,dt. \tag{5.6-4b}$$

Starting with $\mathscr{A}(0,t)$, we determine the Fourier transform $A(0,f)$ by use of (5.6-4b) at $z=0$, then we use (5.6-2) and (5.6-3) to determine $A(z,f)$, from which $\mathscr{A}(z,t)$ is finally composed by using the inverse Fourier transform in (5.6-4a).

This procedure may be simplified by use of the convolution theorem (see Appendix A, Sec. A.1), which provides an explicit expression for $\mathscr{A}(z,t)$ as the convolution $\mathscr{A}(0,t)$ with $h(t)$,

$$\mathscr{A}(z,t) = \int_{-\infty}^{\infty} \mathscr{A}(0,t')h(t-t')\,dt', \tag{5.6-5}$$

where $h(t)$, the impulse-response function, is the inverse Fourier transform of $\mathscr{H}(f)$.

The Slowly Varying Envelope Approximation

Since $\mathscr{A}(z,t)$ is slowly varying in comparison with the central frequency ν_0, the Fourier transform $A(z,f)$ is a narrow function of f with width $\Delta\nu \ll \nu_0$. Such pulses are often called **wavepackets**. To simplify the analysis, we assume that within the frequency range $\Delta\nu$ centered about ν_0, the attenuation coefficient $\alpha(\nu)$ is approximately constant $\alpha(\nu) = \alpha$, and the propagation constant $\beta(\nu) = n(\nu)(2\pi\nu/c_o)$ varies only slightly and gradually with ν, so that it can be approximated by the first three

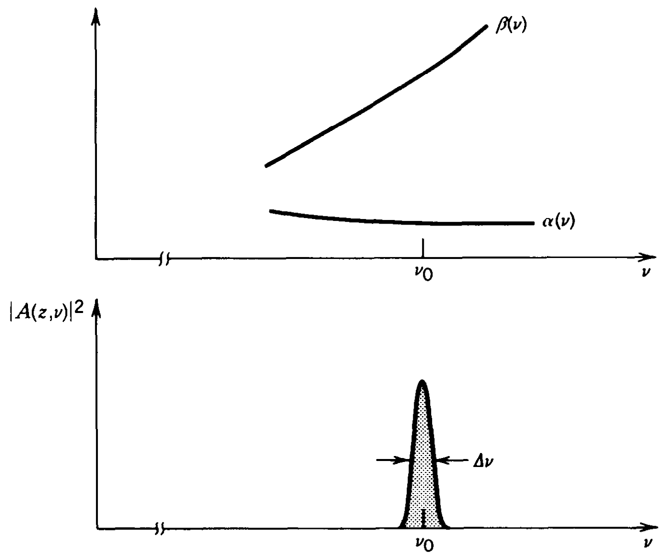

Figure 5.6-2 The attenuation coefficient $\alpha(\nu)$ is assumed to be constant and the propagation constant $\beta(\nu)$ is assumed to be a slowly varying function of ν within the spectral width $\Delta\nu$.

terms of a Taylor series expansion

$$\beta(\nu_0 + f) \approx \beta(\nu_0) + f\frac{d\beta}{d\nu} + \frac{1}{2}f^2\frac{d^2\beta}{d\nu^2}. \tag{5.6-6}$$

Figure 5.6-2 illustrates these functions.

Substituting (5.6-6) into (5.6-3) an approximate expression for the transfer function $\mathcal{H}(f)$ is obtained,

$$\boxed{\mathcal{H}(f) \approx \mathcal{H}_0 \exp(-j2\pi f\tau_d)\exp(-j\pi D_\nu z f^2),} \tag{5.6-7}$$

Approximate Transfer Function

where $\mathcal{H}_0 = e^{-\alpha z/2}$, $\tau_d = z/v$,

$$\boxed{\frac{1}{v} = \frac{1}{2\pi}\frac{d\beta}{d\nu} = \frac{d\beta}{d\omega}} \tag{5.6-8}$$

Group Velocity

and

$$\boxed{D_\nu = \frac{1}{2\pi}\frac{d^2\beta}{d\nu^2} = 2\pi\frac{d^2\beta}{d\omega^2} = \frac{d}{d\nu}\left(\frac{1}{v}\right).} \tag{5.6-9}$$

Dispersion Coefficient

The constants v and D_ν, called the **group velocity** and the **dispersion coefficient,** respectively, are important parameters that characterize the dispersive medium, as we shall see subsequently.

Group Velocity

If the dispersion coefficient is sufficiently small, the third term in the expansion (5.6-6) may also be neglected and $\mathcal{H}(f) \approx \mathcal{H}_0 \exp(-j2\pi f \tau_d)$. The system is then equivalent to an attenuation factor $\mathcal{H}_0 = e^{-\alpha z/2}$ and a time delay $\tau_d = z/v$ (see Appendix A, Sec. A.1, the delay property of the Fourier transform), so that $\mathscr{A}(z,t) = e^{-\alpha z/2}\mathscr{A}(0, t - \tau_d)$. In this approximation the pulse travels at the group velocity v, its intensity is attenuated by the factor $e^{-\alpha z}$, but its initial shape is not altered. By comparison, in an ideal (lossless and nondispersive) medium, $\alpha = 0$ and $\beta(\nu) = 2\pi\nu/c$, so that $v = c$; the pulse envelope travels at the speed of light in the medium and its height and shape are not altered.

Dispersion Coefficient

Since the group velocity $v = 2\pi/(d\beta/d\nu)$ is itself frequency dependent, different frequency components of the pulse undergo different delays $\tau_d = z/v$. As a result, the pulse spreads and its shape is altered. Two identical pulses of central frequencies ν and $\nu + \delta\nu$ suffer a differential delay

$$\delta\tau = \frac{d\tau_d}{d\nu}\delta\nu = \frac{d}{d\nu}\left(\frac{z}{v}\right)\delta\nu = D_\nu z\, \delta\nu.$$

If $D_\nu > 0$ (**normal dispersion**), the travel time for the higher-frequency component is longer than the travel time for the lower-frequency component. Thus shorter-wavelength components are slower, as illustrated schematically in Fig. 5.5-4. Normal dispersion occurs in glass in the visible band. At longer wavelengths, however, $D_\nu < 0$ (**anomalous dispersion**), so that the shorter-wavelength components are faster.

If the pulse has a spectral width σ_ν (Hz), then

$$\boxed{\sigma_\tau = |D_\nu| \sigma_\nu z} \tag{5.6-10}$$

is an estimate of the spread of its temporal width. The dispersion coefficient D_ν is therefore a measure of the pulse time broadening per unit spectral width per unit distance (s/m-Hz).

The shape of the transmitted pulse may be determined using the approximate transfer function (5.6-7). The corresponding impulse-response function $h(t)$ is obtained by taking the inverse Fourier transform,

$$\boxed{h(t) \approx \mathcal{H}_0 \frac{1}{(j|D_\nu|z)^{1/2}} \exp\left[j\pi \frac{(t-\tau_d)^2}{D_\nu z}\right].} \tag{5.6-11}$$

Impulse-Response Function

This may be shown by noting that the Fourier transform of $\exp(j\pi t^2)$ is $\sqrt{j}\, \exp(-j\pi f^2)$ and using the scaling and delay properties of the Fourier transform (see Appendix A, Sec. A.1 and Table A.1-1). The complex envelope $\mathscr{A}(z,t)$ may be obtained by convolving the initial complex envelope $\mathscr{A}(0,t)$ with the impulse-response function $h(t)$, as in (5.6-5).

Gaussian Pulses

As an example, assume that the complex envelope of the incident wave is a Gaussian pulse $\mathscr{A}(0,t) = \exp(-t^2/\tau_0^2)$ with $1/e$ half-width τ_0. The result of the convolution integral (5.6-5), when (5.6-11) is used and $\alpha = 0$, is

$$\mathscr{A}(z,t) = \left[\frac{q(0)}{q(z)}\right]^{1/2} \exp\left[j\pi\frac{(t-\tau_d)^2}{D_\nu q(z)}\right], \tag{5.6-12}$$

where

$$q(z) = z + jz_0, \qquad z_0 = \frac{\pi\tau_0^2}{-D_\nu}, \qquad \tau_d = \frac{z}{v}. \tag{5.6-13}$$

The intensity $|\mathscr{A}(z,t)|^2 = |q(0)/q(z)| \exp[-\pi(t-\tau_d)^2 \operatorname{Im}\{1/D_\nu q(z)\}]$ is a Gaussian function

$$|\mathscr{A}(z,t)|^2 = \frac{\tau_0}{\tau(z)} \exp\left[-\frac{2(t-\tau_d)^2}{\tau^2(z)}\right] \tag{5.6-14}$$

centered about the delay time $\tau_d = z/v$ and of width

$$\boxed{\tau(z) = \tau_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2}.} \tag{5.6-15}$$

Width Broadening of a Gaussian Pulse

The variation of $\tau(z)$ with z is illustrated in Fig. 5.6-3. In the limit $z \gg z_0$,

$$\tau(z) \approx \tau_0\frac{z}{|z_0|} = |D_\nu|\frac{z}{\pi\tau_0}, \tag{5.6-16}$$

so that the pulse width increases linearly with z. The width of the transmitted pulse is

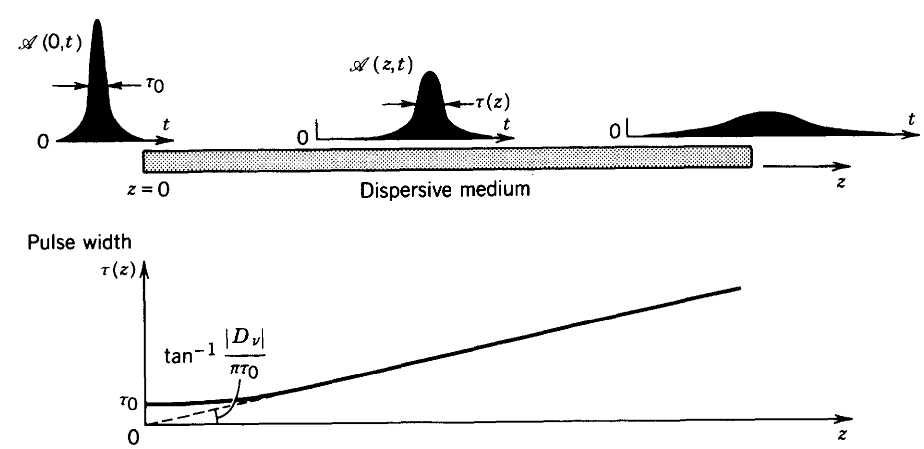

Figure 5.6-3 Gaussian pulse spreading as a function of distance. For large distances, the width increases at the rate $|D_\nu|/\pi\tau_0$, which is inversely proportional to the initial width τ_0.

then inversely proportional to the initial width τ_0. This is expected since a narrow pulse has a broad spectrum corresponding to a more pronounced dispersion. If $\sigma_\nu = 1/\pi\tau_0$ is interpreted as the spectral width of the initial pulse, then $\tau(z) = |D_\nu|\sigma_\nu z$, which is the same expression as in (5.6-10).

*Analogy Between Pulse Dispersion and Fresnel Diffraction

Expression (5.6-11) for the impulse-response function indicates that after traveling a distance z in a dispersive medium, an impulse at $t = 0$ spreads and becomes proportional to $\exp(j\pi t^2/D_\nu z)$, where the delay τ_d has been ignored. This is mathematically analogous to Fresnel diffraction, for which a point at $x = y = 0$ creates a paraboloidal wave proportional to $\exp[-j\pi(x^2 + y^2)/\lambda z]$ (see Sec. 4.1C). With the correspondences x (or y) $\leftrightarrow t$ and $\lambda \leftrightarrow -D_\nu$, the approximate temporal spread of a pulse is analogous to the Fresnel diffraction of a "spatial pulse" (an aperture function). The dispersion coefficient $-D_\nu$ for temporal dispersion is analogous to the wavelength for diffraction ("spatial dispersion"). The analogy holds because the Fresnel approximation and the dispersion approximation both make use of Taylor-series approximations carried to the quadratic term.

The temporal dispersion of a Gaussian pulse in a dispersive medium, for example, is analogous to the diffraction of a Gaussian beam in free space. The width of the beam is $W(z) = W_0[1 + (z/z_0)^2]^{1/2}$, where $z_0 = \pi W_0^2/\lambda$ [see (3.1-8) and (3.1-11)], which is analogous to the width in (5.6-15), $\tau(z) = \tau_0[1 + (z/z_0)^2]^{1/2}$, where $z_0 = \pi\tau_0^2/(-D_\nu)$.

*Pulse Compression in a Dispersive Medium by Chirping

The analogy between the diffraction of a Gaussian beam and the dispersion of a Gaussian pulse can be carried further. Since the spatial width of a Gaussian beam can be reduced by use of a focusing lens (see Sec. 3.2), could the temporal width of a Gaussian pulse be compressed by use of an analogous system?

A lens of focal length f introduces a phase factor $\exp[j\pi(x^2 + y^2)/\lambda f]$ (see Sec. 3.2A), which bends the wavefronts so that a beam of initial width W_0 is focused near the focal plane to a smaller width $W_0' = W_0/[1 + (z_0/f)^2]^{1/2}$, where $z_0 = \pi W_0^2/\lambda$ [see (3.2-13)]. Similarly, if the Gaussian pulse is multiplied by the phase factor $\exp(-j\pi t^2/D_\nu f)$, a pulse of initial width τ_0 would be compressed to a width $\tau_0' = \tau_0/[1 + (z_0/f)^2]^{1/2}$, after propagating a distance $\approx f$ in a dispersive medium with dispersion coefficient D_ν, where $z_0 = -\pi\tau_0^2/D_\nu$. Clearly, the pulse would be broadened again if it travels farther.

The phase factor $\exp(-j\pi t^2/D_\nu f)$ may be regarded as a frequency modulation of the initial pulse $\exp(-t^2/\tau_0^2)\exp(j2\pi\nu_0 t)$. The instantaneous frequency of the modulated pulse ($1/2\pi$ times the derivative of the phase) is $\nu_0 - t/D_\nu f$. Under conditions of normal dispersion, $D_\nu > 0$, the instantaneous frequency decreases linearly as a function of time. The pulse is said to be chirped.

The process of pulse compression is depicted in Fig. 5.6-4. The high-frequency components of the chirped pulse appear before the low frequency components. In a medium with normal dispersion, the travel time of the high-frequency components is longer than that of the low-frequency components. These two effects are balanced at a certain propagation distance at which the pulse is compressed to a minimum width.

*Differential Equation Governing Pulse Propagation

We now use the transfer function $\mathcal{H}(f)$ in (5.6-7) to generate a differential equation governing the envelope $\mathscr{A}(z,t)$. Substituting (5.6-7) into (5.6-2), we obtain $A(z,f) = A(0,f)\exp(-\alpha z/2 - j2\pi f z/v - j\pi D_\nu z f^2)$. Taking the derivative with respect to z, we obtain the differential equation $(d/dz)A(z,f) = (-\alpha/2 - j2\pi f/v - j\pi D_\nu f^2)A(z,f)$. Taking the inverse Fourier transform of both sides, and noting that the inverse Fourier transforms of $A(z,f)$, $j2\pi f A(z,f)$, and $(j2\pi f)^2 A(z,f)$ are $\mathscr{A}(z,t)$, $\partial\mathscr{A}(z,t)/\partial t$, and

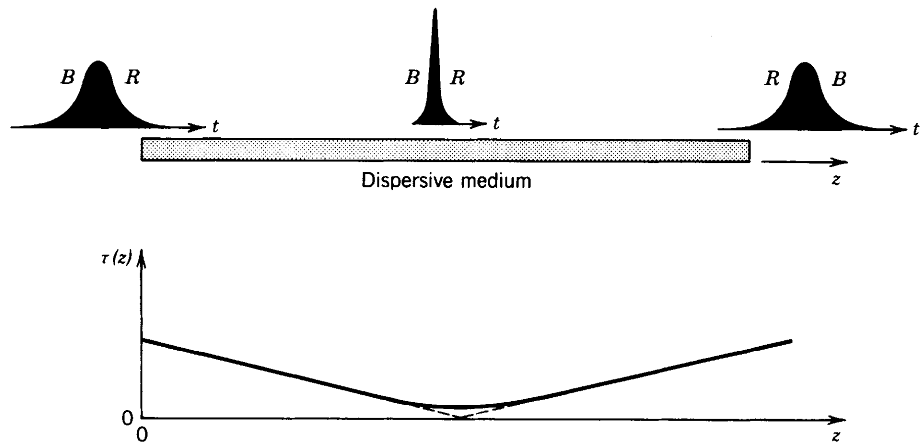

Figure 5.6-4 Compression of a chirped pulse in a medium with normal dispersion. The low frequency (marked R) occurs after the high frequency (marked B) in the initial pulse, but it catches up since it travels faster. Upon further propagation, the pulse spreads again as the R component arrives earlier than the B component.

$\partial^2 \mathscr{A}(z,t)/\partial t^2$, respectively, we obtain a partial differential equation for $\mathscr{A} = \mathscr{A}(z,t)$:

$$\boxed{\left(\frac{\partial}{\partial z} + \frac{1}{v}\frac{\partial}{\partial t}\right)\mathscr{A} + \frac{\alpha}{2}\mathscr{A} - j\frac{D_\nu}{4\pi}\frac{\partial^2 \mathscr{A}}{\partial t^2} = 0.} \tag{5.6-17}$$

Slowly Varying Envelope Wave Equation in a Dispersive Medium

The Gaussian pulse (5.6-12) is clearly a solution to this equation. Assuming that $\alpha = 0$ and using a coordinate system moving with velocity v, (5.6-17) simplifies to

$$\frac{\partial^2 \mathscr{A}}{\partial t^2} + j\frac{4\pi}{D_\nu}\frac{\partial \mathscr{A}}{\partial z} = 0. \tag{5.6-18}$$

Equation (5.6-18) is analogous to the paraxial Helmholtz equation (2.2-22), confirming the analogy between dispersion in time and diffraction in space.

Wavelength Dependence of Group Velocity and Dispersion Coefficient

Since the group velocity v and the dispersion coefficient D_ν are the most important parameters governing pulse propagation in dispersive media, it is useful to examine their dependence on the wavelength. Substituting $\beta = n2\pi\nu/c_o = n2\pi/\lambda_o$ and $\nu = c_o/\lambda_o$ in the definitions (5.6-8) and (5.6-9) yields

$$\boxed{v = \frac{c_o}{N},} \qquad \boxed{N = n - \lambda_o\frac{dn}{d\lambda_o}} \tag{5.6-19}$$

Group Velocity and Group Index

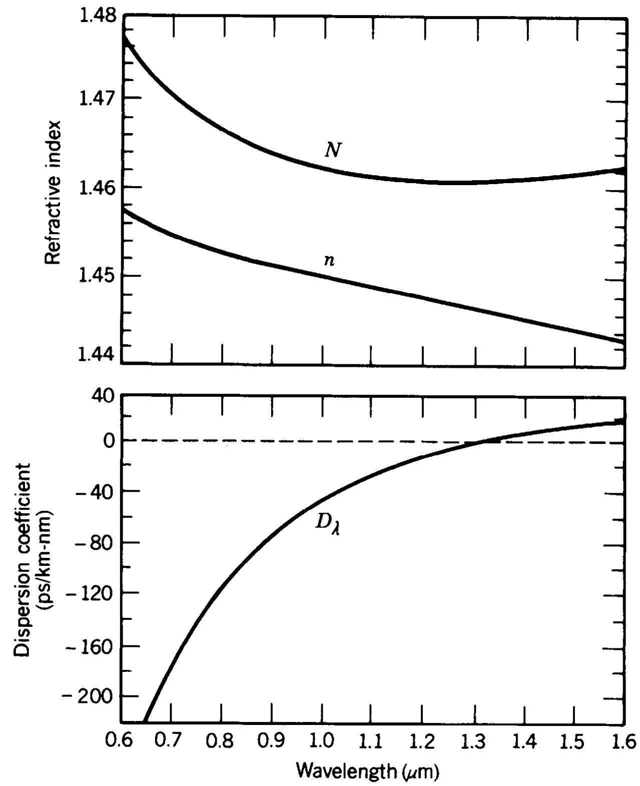

Figure 5.6-5 Wavelength dependence of optical parameters of fused silica: the refractive index n, the group index $N = c_o/v$, and the dispersion coefficient D_λ. At $\lambda_o = 1.312\ \mu\text{m}$, n has a point of inflection, the group velocity v is maximum, the group index N is minimum, and the dispersion coefficient D_λ vanishes. At this wavelength the pulse broadening is minimal.

and

$$\boxed{D_\nu = \frac{\lambda_o^3}{c_o^2}\frac{d^2 n}{d\lambda_o^2}.} \tag{5.6-20}$$

Dispersion Coefficient (s/m-Hz)

The parameter N is often called the **group index**.

It is also common to define a dispersion coefficient D_λ in terms of the wavelength instead of the frequency by use of the relation $D_\lambda\, d\lambda = D_\nu\, d\nu$, which gives $D_\lambda = D_\nu\, d\nu/d\lambda_o = D_\nu(-c_o/\lambda_o^2)$, and†

$$\boxed{D_\lambda = -\frac{\lambda_o}{c_o}\frac{d^2 n}{d\lambda_o^2}.} \tag{5.6-21}$$

Dispersion Coefficient (s/m-nm)

The pulse broadening for a source of spectral width σ_λ is, in analogy with (5.6-10), $\sigma_\tau = |D_\lambda|\sigma_\lambda z$.

†Another dispersion coefficient $M = -D_\lambda$ is also widely used in the literature.

In fiber-optics applications, D_λ is usually given in units of ps/km-nm, where the pulse broadening is measured in picoseconds, the length of the medium in kilometers, and the source spectral width in nanometers. The wavelength dependence of n, N, and D_λ for silica glass are illustrated in Fig. 5.6-5. For $\lambda_o < 1.312$ μm, $D_\lambda < 0$ ($D_\nu > 0$; normal dispersion). For $\lambda_o > 1.312$ μm, $D_\lambda > 0$, so that the dispersion is anomalous. Near $\lambda_o = 1.312$ μm, the dispersion coefficient vanishes. This property is significant in the design of light-transmission systems based on the use of optical pulses, as will become clear in Secs. 8.3, 19.8, and 22.1.

READING LIST

See also the list of general books on optics in Chapter 1.

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PROBLEMS

5.1-1 **An Electromagnetic Wave.** An electromagnetic wave in free space has an electric field $\mathscr{E} = f(t - z/c_o)\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is a unit vector in the x direction, $f(t) = \exp(-t^2/\tau^2)\exp(j2\pi\nu_0 t)$, and τ is a constant. Describe the physical nature of this wave and determine an expression for the magnetic field vector.

5.2-1 **Dielectric Media.** Identify the media described by the following equations, regarding linearity, dispersiveness, spatial dispersiveness, and homogeneity.
(a) $\mathscr{P} = \epsilon_o \chi \mathscr{E} - a\nabla \times \mathscr{E}$,
(b) $\mathscr{P} + a\mathscr{P}^2 = \epsilon_o \mathscr{E}$,
(c) $a_1 \partial^2\mathscr{P}/\partial t^2 + a_2 \partial\mathscr{P}/\partial t + \mathscr{P} = \epsilon_o \chi \mathscr{E}$,
(d) $\mathscr{P} = \epsilon_o\{a_1 + a_2 \exp[-(x^2 + y^2)]\}\mathscr{E}$,
where χ, a, a_1, and a_2 are constants.

5.3-1 **Traveling Standing Wave.** The complex amplitude of the electric field of a monochromatic electromagnetic wave of wavelength λ_o traveling in free space is $\mathbf{E}(\mathbf{r}) = E_0 \sin\beta y \exp(-j\beta z)\hat{\mathbf{x}}$. (a) Determine a relation between β and λ_o.