Material descriptions

Feb. 10, 2014

• Absorption and dispersion in terms of susceptibility (Sect. 5.5):
Electric property measured by permittivity
$\varepsilon = \varepsilon_o(1+\chi)$ and $\chi = \chi' + j\chi''$; real part relates to phase (dispersion) and imaginary part relates to amplitude (absorption) since propagation factor exp(−jkz) has $k = \omega\sqrt{\varepsilon\mu_o}$

• Absorption (attenuation) coef (α) and refractive index n:
$n = 1 + \chi'/2$ and $\alpha = -k_o\chi''$
Note: α and n are functions of freq.

• Measure of absorption and dispersion:
Attenuation coef in dB/km or m^{-1}
Describing freq dependent n -- group velocity
$v_g = d\omega/d\beta = c_o/N$, group index $N = n - \lambda_o dn/d\lambda_o$
where β is wave number.
Material dispersion coef $D_\lambda = -\dfrac{\lambda_o}{c_o}\dfrac{d^2n}{d\lambda_o^2}$ (ps / km-nm).
pulse widening or delay $= |D_\lambda|\Delta\lambda z$ (ps), where $\Delta\lambda$ is linewidth in nm, z is length of fiber in km.

• Kramers-Kronig relations:
Absorption and refractive index are connected by these relations; result of causality. (See Appendix B.1)

• Harmonic oscillator model for media:
Susceptibility is result of a sea of electric dipole

driven by an external electric field.

$d^2P/dt^2 + \sigma dP/dt + \omega_o^2 P = \omega_o^2 \varepsilon_o \chi_o E$ where ω_o is resonant freq, χ_o is dc susceptibility, σ is damping coef and $\vec{P} = \varepsilon_o \chi \vec{E}$

$\chi = \chi_o \dfrac{\nu_o^2}{\nu_o^2 - \nu^2 + j\nu\Delta\nu}$; $\Delta\nu = \sigma/2\pi$ is width of the resonance.

• Modeling dispersive system:

$A(z,t) = F^{-1}[H(f,z)F[A(0,t)]]$ where

$H(f) = \exp(-\alpha(f+\nu_o)z/2 - j[\beta(f+\nu_o) - \beta_o]z)$, ν_o is the carrier (central) freq, f is the modulation freq, β_o is the wave number at carrier freq, $\beta = 2\pi\nu n/c_o$ and $\nu = \nu_o + f$

Note: Assuming $\nu_o \gg f$

HW #2.5 due 2/21/14

1. Problem 5.4-1 (page 192 1st Ed, p. 195 2nd Ed)
2. exercise 3.1-4 (page 91 1st Ed, p. 84 2nd Ed)
3. Find the peak intensity and beam radius of a Gaussian beam by solving paraxial wave equation (Eq. (3.1-2) in the textbook) numerically. More precisely, you will solve the normalized paraxial wave equation $-j\dfrac{\partial A}{\partial Z} + \nabla_\perp^2 A = 0$ where $Z = z/(4z_o)$, $(X, Y) = (x/w_o, y/w_o)$.
Recall the paraxial wave equation can be expressed in Fourier domain as:
$\dfrac{\partial A_F}{\partial Z} = j(K_x^2 + K_y^2)A_F$ where $A_F = FFT(A)$. Now the form of solution of A_F is just that of a 1st order ODE.
Consider a Gaussian beam with $\lambda_o = 1\mu m$ propagating in air has $I_o = 1 W/m^2$ and $w_o = 10\mu m$ at z=0. a) Plot intensity distribution at z=0, b) plot intensity distribution at $z = 2zo$, c) find peak intensity and beam radius at $z = 2zo$. Your results should be obtained from Matlab and compared to analytical solutions.
Matlab function to be used: fft2, ifft2, fftshift, image, colormap(gray(256)). (Hint: to plot 8bit gray scale image, data must be normalized to 0 -255, i.e. input data array of image should be 255*A/max(A).)
4. Exercise 5.5-1 (page 176 in 1st Ed., page 172 in 2nd Ed.)

Extra-Credit

exercise 3.1-5 (page 91 1st Ed, p. 85 2nd Ed)