Pulse propagation Feb. 14, 2014

• A medium generally has multiple resonances. We can generalize the Lorentz oscillator model to multiple frequencies by superposition of models for different resonant frequencies, e.g. Sellmeier Equation

$$n^2 = 1 + \sum_i \chi_{0i} \frac{\nu_i^2}{\nu_i^2 - \nu^2} = 1 + \sum_i \chi_{0i} \frac{\lambda_i^2}{\lambda_i^2 - \lambda^2}$$

where ν_i (λ_i) is the ith resonant frequency (wavelength). Note: Usually, i=3 is enough for freq range of interest, e.g. fused silica has $\lambda_1 = 0.0684\mu m$, $\lambda_2 = 0.1162\mu m$, $\lambda_3 = 9.8962\mu m$ and $\chi_{01} = 0.6962$, $\chi_{02} = 0.4079$, $\chi_{03} = 0.8975$ for $0.21 < \lambda < 3.71\mu m$

• Recall that phase velocity $c = \omega/\beta$ and group velocity $v_g = d\omega/d\beta$.
Or $\beta = \omega/c$ & $\beta' = d\beta/dw = 1/v_g$

• Modeling dispersive system: (Sec. 5.6 1st Ed.)
$A(z,t) = F^{-1}[H(f,z)F[A(0,t)]]$ where
$H(f) = \exp(-\alpha(f+\nu_o)z/2 - j[\beta(f+\nu_o) - \beta_o]z)$, ν_o is the carrier (central) freq, f is the modulation freq, β_o is the wave number at carrier freq, $\beta = 2\pi\nu n/c_o$ and $\nu = \nu_o + f$
Note: Assuming $\nu_o \gg f$

• Approx dispersive system with slowly varying envelope input (i.e. $\nu_o \gg f$):
$\beta(\nu_o + f) \approx \beta_o + f d\beta/d\nu + f^2 d^2\beta/2d\nu^2 + \cdots$

2nd term relates to group velocity $v_g = 2\pi/(d\beta/d\nu)$
3rd term relates to dispersion
D_ν(or β'') $= d^2\beta/2\pi d\nu^2 = dv_g^{-1}/d\nu$ (ps / km-Hz) measure time delay per unit freq and length.
Pulse width $\sigma_\tau = |D_\nu|\sigma_\nu Z$ where z is length of the medium, σ_ν bandwidth of the pulse
Notes: $D_\nu > 0$ normal dispersion -- blue slower red and vice versa.

• Near resonance: Polarity of D_ν switches as freq passing the resonance. N can be less than 1 (fast light) or even be negative (slow light). Only good for a **very short distance**.

• Slow varying envelope eqn for pulse propagation: obtained by inverse Fourier transform of the system eqn in freq domain

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{1}{v_g}\frac{\partial A}{\partial t} + j\frac{D_\nu}{4\pi}\frac{\partial^2 A}{\partial t^2}$$

Analog to diffraction of Gaussian beam -- $x, y \to t$, $\nabla_T^2 \to \partial^2/\partial t^2$ and $\lambda \to -D_\nu$

• Gaussian pulse: In traveling frame without absorption, $\dfrac{\partial A}{\partial z} = j\dfrac{D_\nu}{4\pi}\dfrac{\partial^2 A}{\partial t'^2}$ where $t' = t - z/v_g = t - \tau_d$.
Solution -- $A = A_o\sqrt{q(0)/q(z)}\exp(j\pi t'^2/D_\nu q(z))$ for a pulse at z=0 with $A(0,t) = A_o \exp(-t'^2/\tau_o^2)$ where $q(z) = z + jz_o$, $z_o = \pi\tau_o^2/(-D_\nu)$ and τ_o is pulse width.

Effect of Im(1/(D_νq)) -- pulse width widened
$\tau = \tau_o\sqrt{1+(z/z_o)^2}$; intensity proportional to $1/\tau$.
Effect of Re(1/(D_νq)) -- introduce chirp to the pulse, i.e. various times have different frequency modulation.

• Bandwidth vs linewidth: $\Delta\lambda/\sigma_\nu = -\lambda_o/\nu \rightarrow$ dispersion coef $D_\nu = -D_\lambda \lambda_o/\nu$

• Conductive media: $\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} = (\sigma + j\omega\varepsilon)\vec{E}$ where $\vec{J}$ is current density and σ is conductivity.
Or $\nabla \times \vec{H} = j\omega\varepsilon_e\vec{E}$ where the complex permittivity $\varepsilon_e = \varepsilon + \sigma/(j\omega)$
$\sigma/\omega \gg \varepsilon \rightarrow n - j\alpha/(2k_o) \approx \sqrt{\sigma/(j\omega\varepsilon_o)}$, $n \approx \sqrt{\sigma/(2\omega\varepsilon_o)}$,
$\alpha = \sqrt{2\sigma\omega\mu_o}$ and $\eta \approx (1+j)\sqrt{\omega\mu_o/(2\sigma)}$

• Drude model: $\sigma = \sigma_0/(1 + j\omega\tau)$ where τ is the relaxation time of charges in conductors and σ_0 is low freq conductivity. For $\omega \gg 1/\tau$, $\varepsilon_e \approx \varepsilon\,(1 - \omega^2/\omega_p^2)$ where plasma frequency $\omega_p = \sqrt{\sigma_0/(\varepsilon\tau)}$
$\omega \ll \omega_p$, medium behaves like perfect conductor (all wave reflected)
$\omega \gg \omega_p$, medium behaves like lossless dielectric.
$\omega \approx \omega_p$, n can be very small and the dispersion relation shows a plasmonic band.

• Metamaterials: Man-made composite composite media with resonances in electric and magnetic properties, i.e. both ε and μ can be complex with negative real parts near resonances. Hence, n becomes negative. Such property can be used to construct optical elements that compensate diffraction, e.g. super-lens.

HW #3 due 2/21/14

1. Determine an expression for the group velocity v_g of a resonant medium with refractive index given by the near resonance equations in Eq. (5.5-21), (5.5-19) and (5.5-20) of the 1st Ed. [equivalent to Eq. (5.5-27), (5.5-23) and (5.5-24) of the 2nd Ed.]. Sketch v_g as a function of the frequency ν. (identify regions of normal and anomalous dispersion) [Problem 5.6-2 (page 192) in 1st Ed.]

2. A Gaussian pulse of width $\tau_0 = 100$ ps travels a distance of 1km through an optical fiber made of silica with the characteristics shown in Fig. 5.6-5 (on page 190 in 1st Ed and page 189 in 2nd Ed]. Estimate the time delays τ_d and the width of the received pulse if the wavelength is (a) 0.8 μm, (b) 1.312μm, (c) 1.55μm. [Problem 5.6-3 (page (192)]

Extra-Credit equivalent to 2 problems (2/25/14)

Write a program (in Matlab or other language) to solve $\frac{\partial A}{\partial z} = j \frac{D_\nu}{4\pi} \frac{\partial^2 A}{\partial t'^2}$ [Eq. (5.6-18) in 1st Ed of text book] with the method outlined by $A(z,t) = F^{-1}[H(f,z)F[A(0,t)]]$ where $H(f) = \exp(-\alpha(f+\nu_o)z/2 - j[\beta(f+\nu_o) - \beta_o]z)$, [Eq. (5.6-2) and (5.6-4) in 1st Ed of textbook] and apply FFT. Test your program with the following cases with the amplitude of pulse plotted versus time at z=0, z=0.2km, z=0.4km, z=0.6km, z=0.8km and z=1km : a) Input Gaussian pulse with width = 10ps and wavelength 0.8μm.

b) Input Gaussian pulse with width = 10ps and wavelength 1.55μm.

c) Input Gaussian pulse with width = 1ps and wavelength 0.8μm.

d) Input Gaussian pulse with width = 1ps and wavelength 1.55μm.

Use the dispersion parameter in Fig. 5.6-5 and calculations similar to problem 5.6-3 to guide your calculations.

As a rule of thumb, at least 50 samples falls within the pulse. The entire window should be at least 5 times of the pulse width. This prevents aliasing caused by expansion of the pulse and gives you an idea on total number of samples within the window. Please submit your program with the homework.