Polarization and its model   Feb. 17, 2014

• Polarization:
linear (vertical & horizontal), circular (clockwise & counter) and elliptical polarization referring to the curve tracing out by electric field
Implications -- absorption, refraction and reflection polarization dependent
A carrier can transmit two independent signals
Require (natural or induced) anisotropic medium to make devices, e.g. modulator and polarization converter

• General wave with polarization:
$\vec{E} = \mathrm{Re}\{(\hat{x}A_x + \hat{y}A_y)\exp(j2\pi\nu(t - z/c))\}$ where
$A_x = a_x e^{j\phi_x}$ and $A_y = a_y e^{j\phi_y}$ are complex envelopes.

• Polarization (shape traced out by $\vec{E}$ over time) is determined by $r = a_y / a_x$ and $\phi = \phi_y - \phi_x$, e.g.
Linear -- $\phi_y - \phi_x = 0$ or π
Circular -- $a_y/a_x = 1$ and $\phi_y - \phi_x = \pi/2$ or $-\pi/2$
Elliptic -- in general

• Other geometric representations: Use axis ratio or ellipticity measured by angle χ and the angle ψ between x axis and major axis. $2\chi = 90^{\circ} - \theta$ and $2\psi = \phi$ in a spherical coordination, known as **Poincare sphere**. Its Cartesian axes S_1, S_2 and S_3 forming the Stokes parameters.

• Jones vector: $\vec{J} = (A_x, A_y)^T$ is convenient for modeling polarizing optical elements
On 2D transverse plane -- any polarization can be represented by two unit Jones vectors (basis) $\vec{J}_1$ and $\vec{J}_2$.
$\vec{J}_1$ and $\vec{J}_2$ are orthogonal, i.e. $\vec{J}_1 \cdot \vec{J}_2 = 0$.
2 choices -- linear basis $\vec{J}_1 = (1, 0)^T$ (horizontal) and
$\vec{J}_2 = (0, 1)^T$ (vertical)
circular basis $\vec{J}_1 = (1, j)^T/\sqrt{2}$ (RCP) and
$\vec{J}_2 = (1, -j)^T/\sqrt{2}$ (LCP)
Note: EE convention is just opposite. The textbook follows optics convention.

• Building block for devices: Jones matrix T
Linear polarizer -- filter out one linear polarization.
Wave retarders -- introduce phase difference between linear polarizations, i.e. polarization converter.
Polarization rotator -- preserve polarization but rotate orientation.

• Orientation of polarization devices: New coordinates is rotated by an angle θ from the original $\rightarrow$ $\vec{J}' = R(\theta)\vec{J}$ in terms of the new coordinates where $R(\theta)$ is the rotational transformation matrix.
Input (original) + a device (new) $\rightarrow$ output (original).
Steps -- use $R(\theta)$ to transform input Jones vector (new), then process the input by Jones matrix to obtain output Jones vector (new) and invert output back to original system

• Polarization dep. reflection and refraction:
Slab at y=0 with different refraction index
X axis is perpendicular (TE or s polarization) to plane of incidence and y is parallel to plane of incidence (TM or p polarization).
Note that input $\vec{J}_1$, transmitted $\vec{J}_2$ and reflected $\vec{J}_3$ do not have the same y axes.
$\vec{J}_2 = \underline{t}\vec{J}_1$ and $\vec{J}_3 = \underline{r}\vec{J}_1$

$$\underline{r} = \begin{bmatrix} r_x & 0 \\ 0 & r_y \end{bmatrix}, \underline{t} = \begin{bmatrix} 1 + r_x & 0 \\ 0 & 1 + r_y \end{bmatrix}$$

• Power reflectance: $R = |r|^2$
Normal incidence $R = (n_1 - n_2)^2/(n_1 + n_2)^2$

Anisotropic materials

Feb. 17, 2014

• Solid state media (Sec. 6.3):
Material with crystalline structure used to make polarization devices.

• Representation:
Align electric field E along principle axes $\rightarrow \underline{\varepsilon}$ is a diagonal matrix
Three different elements - biaxial
Two different elements - uniaxial; optic axis is z (3) axis and $n_3 = n_e$; $n_e > n_o$ positive uniaxial.
$\underline{\eta} = \varepsilon_o \underline{\varepsilon}^{-1}$ is impermeability tensor; use $\underline{\eta}$ to associate

the medium as an index ellipsoid $\eta_{ij}x_i x_j = 1$ or $(x_1/n_1)^2 + (x_2/n_2)^2 + (x_3/n_3)^2 = 1$.

• Wave propagation:
Propagate along a principal axis polarized along a principal axis -- $k_i = n_i k_o$ with i=1,2 or 3 and polarization preserved.
Propagate along a principal axis -- polarization changed owing to phase difference between E along the two principal axes.
Arbitrary propagation direction -- rely on graphical method or matrix calculations

• Graphical method:
Draw a index ellipsoid which intersect x at n_1, y at n_2 and z at n_3.
Slice across the ellipsoid with a knife perpendicular to propagation direction $\rightarrow$ cross section is a ellipse.
Major (minor) axis give the refractive index n´ (n´´)for the electric flux density or displacement $\vec{D}'$ ($\vec{D}''$) along the same direction.

• Analytic method:
Assume plane wave $\vec{e}_i = \vec{E}\cos(\omega t - \vec{k}\cdot\vec{r})$ or in phaser form $\vec{E}_i = \vec{E}\exp(j(\omega t - \vec{k}\cdot\vec{r}))$ in source free media
$\nabla\cdot\vec{d}_i = 0 \rightarrow \vec{k}\cdot\vec{D} = 0$; $\nabla\times\vec{e}_i = -\partial\vec{b}/\partial t$
$\rightarrow \vec{k}\times\vec{E} = \omega\mu_o\vec{H}$; $\nabla\times\vec{h}_i = \partial\vec{d}/\partial t \rightarrow \vec{k}\times\vec{H} = -\omega\vec{D}$
$\vec{D}$ and $\vec{E}$ on the same plane but not parallel!

Wave eqn -- $\vec{k} \times (\vec{k} \times \vec{E}) = -k_o^2 \vec{D}/(\varepsilon_o)$

Solution for $\vec{D}$ -- $D_j = u_j(\hat{k} \cdot \vec{E})/(\varepsilon_j^{-1} - \varepsilon_o^{-1} n^{-2})$ where

$\hat{k} = \vec{k}/|\vec{k}| = (u_1, u_{2,} u_3)$

Solution for n -- Fresnel's eqn

$u_1^2/(n^{-2} - n_1^{-2}) + u_2^2/(n^{-2} - n_2^{-2}) + u_3^2/(n^{-2} - n_3^{-2}) = 0$

Two solutions for n (n´ and n´´); for n´, get direction $\hat{D}$´; for n´´, direction $\hat{D}$´´. Decompose

$\vec{D}_i = d´\hat{D}´ + d´´\hat{D}´´$.

Please read **pp. 49-53 in Ch. 3 of the supplementary notes**.

HW #4 due 2/28/14

1) Problem 6.1-4 in 1st Ed. page 235 (Problem 6.1-8 in 2nd Ed. page 241)

2) An optical isolator transmits light traveling in one direction and blocks it in opposite direction. Show that isolation of the light reflected by a planar mirror may be achieved by using a combination of a linear polarizer and a quarter-wave retarder with axes at 45^o with respect to the transmission axis of the polarizer. (Problem 6.1-6 in 1st Ed.)

3) Problem 6.2-2 in 1st Ed. page 236 (note: θ_1 plus θ_2 equals to 90^o at the Brewster angle) (Problem 6.2-5 in 2nd Ed. page 241)