Ray, Energy and Optical Activity Feb. 24, 2014

• k surface: Give the value of k or n in certain propagation direction at certain angular freq ω.
E.g. in uniaxial crystal, the $|\vec{k}| = n_o k_o$ when $\vec{k}$ is along optic axis and called ordinary wave.
For $\vec{k}$ making an angle θ with the optic axis, there are 2 wave polarizations discriminated by the medium.
Ordinary wave on x-y plane
Extra-ordinary wave which has z component and $n(\theta)$ given by $\cos^2\theta n_o^{-2} + \sin^2\theta n_e^{-2} = n^{-2}$.
Note - k surface **not equal to** index ellipsoid.

• Ray and energy in same direction: Ray and Poynting vector perpendicular to k surface, i.e.
$\vec{S}/|\vec{S}| = \nabla k_o / |\nabla k_o|$ where gradient is taken with respect to k_1, k_2 and k_3.
$\vec{k}$ may not be parallel to ray.

• Snell's law in anisotropic media: Source (incident) and responses (reflected & refracted) must match phase in the co-propagating direction at the origin;
Relation between source in medium 1 and transmitted waves in medium 2 -- $k_o \sin\theta_1 = k(\theta_2)\sin\theta_2$ where θ_1 is incident angle and θ_2 is refracted angle.
Different polarizations (TE, TM) have different refractive indices dependent on $\theta_1 \rightarrow$ birefringence; occurs even in normal incidence.

Example of graphical method in solving refraction problem in anisotropic media with Snell's law.

• Optical activity: Optical active materials operate on circular polarizations and act like rotators,.e.g. sugar and its mirror image.
They introduce different phase shift to LCP and RCP. rotation power $\rho = \pi(n_{-} - n_{+})/\lambda_o$ where n_+ for RCP and n_- for LCP.

• Medium model: spatial dispersive and non-local; $\vec{D} = \varepsilon\vec{E} + \varepsilon_o \xi j\omega\vec{B}$ where ξ is a constant measuring helicity of the medium.
$n_{\pm} = (n_o^2 \pm |\vec{G}|)^{1/2}$ where $\vec{G} = \xi\vec{k}$ is the gyration vector.

Faraday effect and LC

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• Faraday effect: Another principle for rotator
Different from optical active media - the rotation reference axis depends on the magnetic field instead of propagation direction.
Rotation power $\rho = \mathrm{VB}$ where B magnetic flux density and $\mathrm{V} = -\pi\gamma/(\lambda_o n_o)$ is the Verdet constant and γ is magnetogyration coef.
Device - optical isolator based on nonreciprocity of Faraday effect.

• Liquid crystal: Nematic, Smectic and Cholesteric
Twisted nematic LC - anisotropic medium with its optic axis rotates along propagation direction z; twisted angle $\theta = \alpha z$ and phase retardation $\beta = (n_e - n_o)k_o$ where α is the twist coef.
Usually $\beta \gg \alpha$, the LC can be considered as a rotator cascaded with a phase retarder.

• Polarization devices: Selective absorption (polaroid), selective reflection (Brewster angle), selective refractive (anisotropic crystal).
Modulator - retarder or rotator sandwiched by cross polarizers where the phase retardation or rotation angle is controlled by a signal.
Possible media - electro-optic crystal, LC.
Isolator - Faraday effect

• Simple optical components (Sect. 2.4): Reflection

(mirror); refraction or transmission (lens)
Ideal transmission of device - $t = h_o \exp(-j(n-1)k_o d)$;
E.g. lens $t \approx h_o \exp(jk_o(x^2 + y^2)/2f)$
Diffraction gratings - d is a periodic function of transverse dimension x. Splitting angle $\theta_q = \theta_i + q\lambda/\Lambda$ where q is the diffraction order and Λ is the period of variation in d
Note - the angle is wavelength dependent and above description valid for $\Lambda \gg \lambda$
Inhomogeneous index - grade-index components

• Interference: 2 **coherent** beams,
$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos\phi$ where $\phi = 0$ or $m\pi$ (m even) constructive interference and $\phi = m\pi$ (m odd) destructive interference.
2 incoherent beams, $I = I_1 + I_2$
Devices: interferometers (Mach-Zehnder, Michelson, Sagnac), optical cavity.
Applications: phase detection (translate phase info into amplitude info), making diffraction grating or hologram, laser.

HW #5 due 3/7/14
1. A half-wave plate has a phase retardation of $\Gamma = \pi$. Assume that the plate is oriented so that the azimuth angle (i.e. the angle between the x axis and the slow axis of the plate) is ψ.
a. Find the polarization state of the transmitted beam, assuming that the incident beam is linearly polarized in the y direction.
b. Show that a half-wave plate will convert right circularly polarized light into left circularly polarized light, and vice versa, regardless of the azimuth angle of the plate.
c. Lithium tantalate ($LiTaO_3$) is a uniaxial crystal with $n_o = 2.1391$ and $n_e = 2.1432$ at $\lambda = 1\mu m$. Find the half-wave thickness at this wavelength, assuming the plate is cut in such a way that the surfaces are perpendicular to the x axis of the principal coordinates (i.e. x-cut). (notes: use Jones vector for a.

and b.)
2. Problem 6.3-2 (p. 236 in 1st Ed., p. 241 in 2nd Ed.)
3. Problem 6.6-2 (p. 237 in 1st Ed., p. 242 in 2nd Ed.)
4. Show that if $G \ll n_o$, the rotatory power of an optically active medium (rotation of the polarization plane per unit length) is approximately given by $\rho = -\pi G/(\lambda_o n_o)$ [Exercise 6.4-1 (p. 225 in 1st Ed.)]
Extra-Credit
6.3-3 (Note: The angle between the optic axis and the boundary is 30^o. Refer to Fig. 6.3-13 to visualize the problem.) (p. 236 in 1st Ed., p. 242 in 2nd Ed.)