# Electrons Vs Photons

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• Linear Vs nonlinear transfer characteristics of devices

• Digital devices: switches, gates, trigger
Electronics -- transistor; electrical input / output & electrical control
Photonics -- spatial light modulator, e.g. liquid crystal, photorefractive medium (electro-optics effect); electrical control & optical input / output
all optical switch, e.g. self-focusing, self-steering, limiter; optical input, output and control

• Analog devices: mixer, filter, amplifier
Electronics -- diode mixer, square law detector
Photonics -- harmonic generation, four wave mixing, down conversion, up conversion and parametric amplification

• Enemy of high speed communication: dispersion or channel fading

• Result of canceling dispersion with nonlinearity: Solitons
A pulse that does not decay after long distance propagation in nonlinear medium, e.g. optical fiber

• Time domain vs frequency domain representation of pulses
Numerical tool: Fast Fourier Transform (FFT)

Efficient for # of sampling points = $2^n$

Sampling freq $1/\Delta$ ($\Delta$ sampling interval) must be twice of the max. freq $f_c$

Fourier transform of h(t) at freq $f_n = H(f_n) \approx \Delta H_n$ where $H_n$ is the FFT of h at the $n^{th}$ sampling point

• Wave propagation in free space for time varying fields:

**Notation -- lower case letters to denote time varying fields**

$\nabla \times \vec{h} = \varepsilon_o \partial\vec{e}/\partial t$, $\nabla \times \vec{e} = -\mu_o \partial\vec{h}/\partial t$, $\nabla \cdot \vec{e} = 0$, $\nabla \cdot \vec{h} = 0$

$\vec{h}$ -- magnetic field intensity (A/m), $\vec{e}$ -- electric field intensity (V/m)

Effects of material (polarization $\vec{p}$, magnetization $\vec{m}$):

$\vec{d} = \varepsilon_o \vec{e} + \vec{p}$, $\vec{b} = \mu_o \vec{h} + \mu_o \vec{m}$

Wave equation: $\nabla^2 \vec{e} - \partial^2 \vec{e}/(c_o^2 \partial t^2) = 0$

where $c_o = 3 \times 10^8$m/s is the speed of light in vacuum.

Boundary conditions: $b_{1n} = b_{2n}$, $d_{1n} = d_{2n}$, $e_{1t} = e_{2t}$, $h_{1t} = h_{2t}$

Power carried by wave: instantaneous Poynting Vector $\vec{\mathbf{p}} = \vec{e} \times \vec{h}$(W/m$^2$) which measures intensity.

# Electromagnetic wave

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• Application of Maxwell equations:

$\vec{e} \rightarrow \vec{h}$ by $\nabla \times \vec{e} = -\mu_o \partial \vec{h}/\partial t$

$\vec{h} \rightarrow \vec{e}$ by $\nabla \times \vec{h} = \varepsilon_o \partial \vec{e}/\partial t$

• EM Wave in air:

$\vec{e}(\vec{r}, t) = \hat{y}[A(\vec{r}, t)g(k(c_o t - \vec{r} \cdot \hat{k})) + B(\vec{r}, t)g(k(c_o t + \vec{r} \cdot \hat{k}))]$

where position vector $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$.

Description -- y polarization, A(.) & B(.) envelops,

g(.) propagation factor of the carrier,

$k = \omega/c_o = 2\pi\nu/c_o = 2\pi/\lambda$ wave number, $\lambda$ wavelength,

$c_o = 1/\sqrt{\varepsilon_o \mu_o}$ speed of light

Propagation direction of envelop A -- $+\hat{k}$

Propagation direction of envelop B -- $-\hat{k}$

• Medium description: $\vec{p} = \varepsilon_o \chi \vec{e}$

Linear, nondispersive, homogeneous, isotropic and $\mu = \mu_o \rightarrow \chi$ is constant and $c = c_o/n$ where $n = \sqrt{\varepsilon/\varepsilon_o} = \sqrt{1 + \chi}$ is refractive index.

Inhomogeneous $\rightarrow \chi(r)$ is a function of space.

Anisotropic $\rightarrow \chi$ is a matrix and $\vec{p}$ depends on orientation of $\vec{e}$

Dispersive $\rightarrow \vec{p}$ does not respond instantaneously and depends on previous values of $\vec{e}$, i.e. the system has memory $\rightarrow$ require convolution to model $\vec{p}$.

In freq domain, $\chi$ is freq dependent.

Nonlinear $\rightarrow \vec{p}$ is a nonlinear function of $\vec{e}$

• Monochromatic wave (time harmonic or phasor):

**Notation -- upper case letters to denote phasors**

$\nabla \times \vec{H} = j\omega\vec{D}$, $\nabla \times \vec{E} = -j\omega\vec{B}$, $\nabla \cdot \vec{D} = 0$, $\nabla \cdot \vec{B} = 0$.

complex Poynting vector $\vec{S} = \vec{E} \times \vec{H}^* / 2$

average Poynting vector $\vec{S}_{ave} = \mathrm{Re}\{\vec{S}\}$

Application of average Poynting vector :- an EM wave carries (linear and angular) momentum that can put **radiation pressure** on objects, e.g. small particles. Average rate of momentum over a cross section area = $\vec{S}_{ave}/c$; Average rate of angular momentum = $\vec{r} \times \vec{S}_{ave}/c$

- Various forms of wave:

Plane wave (far field) -- $|\vec{E}| = |\vec{H}|\eta$ and $\hat{k} = \hat{E} \times \hat{H}$ where $\eta = \sqrt{\mu/\varepsilon}$ is wave impedance. $\vec{S}_{ave} = \hat{k}|E_o|^2/(2\eta)$

Spherical wave (near field) (see Sect. 2.2)

Paraboloidal wave or Gaussian beam (Fresnel approximation) (see Sect. 3.1)

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HW #1 due 2/11/14

1. Problem 5.1-1 An electromagnetic wave in free space has an electric field $\vec{e} = f(t - z/c_o)\hat{x}$ where $\hat{x}$ is a unit vector in the x direction, $f(t) = \exp(-t^2/\tau^2)\exp(j2\pi\nu_o t)$ and $\tau$ is a constant. Describe the physical nature of this wave and determine an expression for the magnetic field vector. (physical nature in the problem refers to polarization, propagation direction, envelop, propagation factor and so on) (page 191)
2. i) Use matlab to plot the amplitude of the Fourier transform of the output versus frequency of the following systems and inputs. Comments of the frequency of each component and the relative amplitude of different frequency components

a) a square law device $i = v^2$ with input v = $\cos(2\pi 50t) + \cos(2\pi 120t)$

b) a diode mixer with transfer characteristics $i = i_o(\exp(ev/kT) - 1)$ where $i_o = 10$pA is the reverse bias current, $kT/e = 26$mV at room temperature. The input is $v = 0.3 + 0.2\cos(2\pi 50t)$V, i.e. dc biasing of 0.3V and sinusoidal modulation of amplitude 0.2V.

Extra-Credit

For question 2b, estimate the ratio of the amplitude of 2nd and 3rd harmonics by considering the square and cubic terms in the Taylor series expansion of the transfer characteristics. Compare your estimate to the numerical result in 2b.