Interference and resonators Mar. 3, 2014

• Multiple wave interference: Coherent interference of M waves with phase increased by ϕ for each wave, $I = I_o \sin^2(M\phi/2)/\sin^2(\phi/2)$. Typical result of bouncing of laser beam between two surfaces → fringes. Incoherent, $I = I_o M$, e.g. result of using M light bulbs.

• Interferometer with loss: Resonant in an interferometer is measured by Finesse, F (corresponding to Q in RLC) In an interferometer, waves are reflected many times. Each reflection reduced power by r^2, $I = I_{max}/[1 + (2F/\pi)^2 \sin^2(\phi/2)]$ where $F = \pi\sqrt{r}/(1 - r)$ Observations -- resonance width $(\Delta\phi)_{FWHM} = 2\pi/F$; ϕ relates to the round trip delay.

• Application: Holograms (Sect 4.5) -- record 3D information (phase and amplitude) with interference pattern. When uniform plane wave illuminates the pattern, 3D image results

Transmittance or reflectance of a hologram $\approx I_r + I_o + 2\sqrt{I_r I_o}\cos(\phi_r - \phi_o)$ result of interference of object and record beams

Reconstruct beam $\approx I_r + I_o + \sqrt{I_r}U_o + \sqrt{I_r}U_o{}^*$

Note: record and playback beams indep of space (uniform), 3rd term desired image, 4th term conjugate image.

• Drawbacks: Difficulties in isolate the desired image;

playback required laser.
Off-axis hologram -- different angles for playback, image and conjugate beams, i.e. image can be extracted by spatial filtering with a pin hole.
Volume Hologram -- playback with ordinary light; require a thick film and $I = I_r + I_o + 2\sqrt{I_o I_r}\cos(\vec{k}_g \cdot \vec{r})$ where $\vec{k}_g = \vec{k}_o - \vec{k}_r$ describing a grating with period $\Lambda = 2\pi/|\vec{k}_g|$.
Image or conjugate can be reconstructed depending on incident angle of the playback light which is selective reflected by the grating.
Function as 3D ROM

- Planar-mirror resonator (Sect. 9.1 1st Ed. Sec. 10.1 2nd Ed.): Also called Fabry-Perot etalon -- free spectral range $V_F = c/2d$ where d is the separation of mirrors
Output freq is multiple of V_F
Density of mode (# / freq - length) $= 2/V_F d$; resonance width $\delta\nu = V_F/F$

- Attenuation in resonator: $r^2 = R_1 R_2 e^{-2d\alpha_s}$ and total attenuation coef $\alpha_r = \alpha_s + \alpha_m$; $F \approx \pi/\alpha_r d$
Photon lifetime -- (loss / round trip)$^{-1} = 1/c\alpha_r$

- Spectral filter: found in monochromator and laser; requires narrow resonance width and wide V_F with freq tuning by adjusting d

Electro-optics (Ch. 18 1st Ed, Ch. 20 2nd Ed)

- Electro-optic effect: Control of refractive index with **low** freq electric field (˜GHz)
$n(E) = n + a_1 E + a_2 E^2 + \cdots$
In terms of impermeability η,
$n(E) = n - rn^3 E/2 - sn^3 E^2/2 + \cdots$ where
$\eta(E) = \eta + rE + sE^2 + \cdots$
2nd term Pockel effect requires certain symmetry only found in crystal, e.g. $LiNb0_3$ $r \approx 10^{-12} - 10^{-10} m/V$
3rd term Kerr effect can occur in gas and liquids, e.g. for crystal $s \approx 10^{-18} - 10^{-14} m^2/V^2$

- Device principles: based on phase modulation with low freq electric field
Result in change of polarization or amplitude (interferometer)

- Merit for phase modulation: half wave voltage V_π -- voltage to cause a π phase shift, i.e. voltage difference between on and off state.
$V_\pi = d\lambda_o/(Lrn^3)$ where L is the optical path, d is the distance between electrodes
config -- longitudinal → high voltage;
transverse → larger electric circuit RC delay time where bandwidth $\approx 1/2\pi RC$
traveling wave → match the velocity of light with velocity of modulating field and reduce transit time

$T = L/v$ which limits modulation bandwidth to 1/T.

- Dynamic retarder: $V_\pi = d\lambda_o/[L(r_1 n_1^3 - r_2 n_2^3)]$

- Intensity modulator: Interferometer with 50/50 beam splitter -- $T = \cos^2(\phi/2)$ where $\phi = \phi_o - \Delta\phi$

Analog req linear operation, we choose $\phi_o = \pi/2$

Digital, we choose $\phi_o = m2\pi \rightarrow$ T=1 at V=0 and T=0 at $V = V_\pi$

Cross polarizers -- $T = \sin^2(\Gamma/2)$

- Scanners: Transmission angle for a prism $\theta = (n-1)\alpha$; change in scan angle $\Delta\theta = -\alpha r n^3 E/2$

Resolution $N \approx V/(2V_\pi)$; very high voltage

Use the combination of electric controlled rotator and polarizing filter to achieve spatial switching.

HW #6 due 3/14/14 (no late HW)

1. Exercise 2.4-5 (p. 61 in 1st Ed.; page 56 in 2nd Ed.)
2. Exercise 2.5-1 (page 67 in 1st Ed.; page 61 in 2nd Ed.) and 2.5-3 (page 69 in 1st Ed.; page 64 in 2nd Ed.)
3. Exercise 9.1-1 (page 315 in 1st Ed.; Exercise 10.1-1 a on page 371 in 2nd Ed.) and exercise 9.1-2 (page 320 in 1st Ed.; Exercise 10.1-2 on page 375 in 2nd Ed.)
4. Problem 9.1-2 and also find the Q factor. Assuming that the free-space wavelength of the generated light is 1.55 μm, estimate the longitudinal mode number q. (page 340 in 1st Ed.; problem 10.1-4 on page 400 in 2nd Ed.)

Extra-Credit

Problem 4.5-1 (page 155)