Interference and resonators

• Multiple wave interference: Coherent interference of M waves with phase increased by $\phi$ for each wave, $I = I_0 \sin^2(M\phi/2)/\sin^2(\phi/2)$. Typical result of bouncing of laser beam between two surfaces $\rightarrow$ fringes. Incoherent, $I = I_0 M$, e.g. result of using M light bulbs.

• Interferometer with loss: Resonant in an interferometer is measured by Finesse, $F$ (corresponding to $Q$ in RLC) In an interferometer, waves are reflected many times. Each reflection reduced power by $r^2$, $I = I_{\text{max}}/[1 + (2F/\pi)^2 \sin^2(\phi/2)]$ where $F = \pi \sqrt{r}/(1 - r)$
Observations -- resonance width ($\Delta \phi$) FWHM $= 2\pi/F$; $\phi$ relates to the round trip delay.

• Application: Holograms (Sect 4.5) -- record 3D information (phase and amplitude) with interference pattern. When uniform plane wave illuminates the pattern, 3D image results
  Transmittance or reflectance of a hologram
  $\approx I_r + I_o + 2\sqrt{I_rI_o} \cos(\phi_r - \phi_o)$ result of interference of object and record beams
  Reconstruct beam $\approx I_r + I_o + \sqrt{I_rU_o} + \sqrt{I_rU_o}^*$
  Note: record and playback beams indep of space (uniform), 3rd term desired image, 4th term conjugate image.

• Drawbacks: Difficulties in isolate the desired image;
playback required laser.
Off-axis hologram -- different angles for playback, image and conjugate beams, i.e. image can be extracted by spatial filtering with a pin hole.
Volume Hologram -- playback with ordinary light; require a thick film and $I = I_r + I_o + 2\sqrt{I_o I_r} \cos (\vec{k}_g \cdot \vec{r})$ where $\vec{k}_g = \vec{k}_o - \vec{k}_r$ describing a grating with period $\Lambda = 2\pi/|\vec{k}_g|$.
Image or conjugate can be reconstructed depending on incident angle of the playback light which is selective reflected by the grating.
Function as 3D ROM

• Planar-mirror resonator (Sect. 9.1 1st Ed.  Sec. 10.1 2nd Ed.): Also called Fabry-Perot etalon -- free spectral range $V_F = c/2d$ where d is the separation of mirrors
Output freq is multiple of $V_F$
Density of mode (# / freq - length) = $2/V_F d$; resonance width $\delta \nu = V_F / F$

• Attenuation in resonator: $r^2 = R_1 R_2 e^{-2d\alpha_s}$ and total attenuation coef $\alpha_r = \alpha_s + \alpha_m$; $F \approx \pi/\alpha_r d$
Photon lifetime -- (loss / round trip)$^{-1} = 1/c\alpha_r$

• Spectral filter: found in monochromator and laser; requires narrow resonance width and wide $V_F$ with freq tuning by adjusting d
-3-

Electro-optics (Ch. 18 1st Ed, Ch. 20 2nd Ed)

- Electro-optic effect: Control of refractive index with low freq electric field (~GHz)

\[ n(E) = n + a_1 E + a_2 E^2 + \cdots \]

In terms of impermeability \( \eta \),

\[ n(E) = n - r n^3 E/2 - s n^3 E^2/2 + \cdots \]

where

\[ \eta(E) = \eta + r E + s E^2 + \cdots \]

2nd term Pockel effect requires certain symmetry only found in crystal, e.g. LiNbO\(_3\) \( r \approx 10^{-12} - 10^{-10} \text{ m/V} \)

3rd term Kerr effect can occur in gas and liquids, e.g. for crystal \( s \approx 10^{-18} - 10^{-14} \text{ m}^2/\text{V}^2 \)

- Device principles: based on phase modulation with low freq electric field

Result in change of polarization or amplitude (interferometer)

- Merit for phase modulation: half wave voltage \( V_\pi \)-- voltage to cause a \( \pi \) phase shift, i.e. voltage difference between on and off state.

\[ V_\pi = d \lambda_o / (L n^3) \]

where \( L \) is the optical path, \( d \) is the distance between electrodes

config -- longitudinal \( \rightarrow \) high voltage;

transverse \( \rightarrow \) larger electric circuit RC delay time

where bandwidth \( \approx 1/2\pi RC \)

traveling wave \( \rightarrow \) match the velocity of light with velocity of modulating field and reduce transit time
\[ T = \frac{L}{v} \] which limits modulation bandwidth to \( \frac{1}{T} \).

- **Dynamic retarder:** \( V_\pi = d\lambda_o/[L(r_1 n_1^3 - r_2 n_2^3)] \)

- **Intensity modulator:** Interferometer with 50/50 beam splitter -- \( T = \cos^2(\phi/2) \) where \( \phi = \phi_o - \Delta \phi \)
  Analog req linear operation, we choose \( \phi_o = \pi/2 \)
  Digital, we choose \( \phi_o = m2\pi \rightarrow T=1 \) at \( V=0 \) and \( T=0 \) at \( V = V_\pi \)
  Cross polarizers -- \( T = \sin^2(\Gamma/2) \)

- **Scanners:** Transmission angle for a prism \( \theta = (n - 1)\alpha \); change in scan angle \( \Delta \theta = -\alpha n^3 E/2 \)
  Resolution \( N \approx V/(2V_\pi) \); very high voltage
  Use the combination of electric controlled rotator and polarizing filter to achieve spatial switching.

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HW #6 due 3/14/14 (no late HW)
1. Exercise 2.4-5 (p. 61 in 1st Ed.; page 56 in 2nd Ed.)
2. Exercise 2.5-1 (page 67 in 1st Ed.; page 61 in 2nd Ed.) and 2.5-3 (page 69 in 1st Ed.; page 64 in 2nd Ed.)
4. Problem 9.1-2 and also find the Q factor. Assuming that the free-space wavelength of the generated light is 1.55 \( \mu m \), estimate the longitudinal mode number \( q \). (page 340 in 1st Ed.; problem 10.1-4 on page 400 in 2nd Ed.)
Extra-Credit
Problem 4.5-1 (page 155)