HW 10 Solution

1) 2.15 $\lambda_o = 0.6328\ \mu m$, $\Delta\nu_D = 1.5\,GHz$, $d = 0.3\,m$, $n=1$.

$A_{21} = \frac{1}{20\times10^{-3}}\ (s^{-1})$, $N_2 = 10^{16}\ cm^{-3}$, $N_1 = 5\times10^{14}\ cm^{-3}$.

$g_2 = 3$, $g_1 = 5$, Gaussian line $\Rightarrow g(\nu_o) = \frac{2}{\Delta\nu_D}\sqrt{\frac{\ln 2}{\pi}}$

a) $\sigma_{21}(\nu_o) = \frac{A_{21}}{8\pi}\frac{\lambda_o^2}{n^2}g(\nu_o) = \frac{(0.6328\times10^{-6})^2\times 2}{20\times10^{-3}\,8\pi\times1.5\times10^{9}}\sqrt{\frac{\ln 2}{\pi}}$

$= 4.989\times10^{-22}\,(m^2) = 4.989\times10^{-18}\,(cm^2)$

b) $\gamma(\nu_o) = \sigma_{12}(\nu_o)\left(N_2 - \frac{N_1}{g_1}g_2\right) = 4.989\times10^{-18}\left(10^{16} - \frac{3}{5}10^{14}\times5\right)$

$= 0.04833\ (cm^{-1})$

c) $G_{sp} = e^{\gamma(\nu_o)d} = e^{0.04833\times30} = 4.263$

2) Exercise 14.1-1

a) $\lambda_o = 694.3\,nm$, $\Delta\nu = 330\,GHz$, $t_{sp} = 3\,ms$, $n = 1.76$

$N_1 + N_2 = 10^{22}\,cm^{-3}$, $T = 300k$

$\frac{N_2}{N_1} = e^{-h\nu/kT} \Rightarrow \frac{N_2}{N_1} = e^{-\frac{6.63\times10^{-34}\times3\times10^{8}/694.3\times10^{-9}}{1.38\times10^{-23}\times300}} = e^{-69.22}$

(Note: Another way to comput $h\nu/kT$: $E = h\nu = \frac{1.24}{0.6943}(ev)$

& $kT = 26\times10^{-3}(ev)$ at $300K$; $\frac{E}{kT} = 68.69$)

$\frac{N_2}{N_1+N_2} = \frac{e^{-69.22}}{1+e^{-69.22}} \Rightarrow \frac{N_2}{N_1+N_2} \simeq e^{-69.22} = 8.672\times10^{-31} \Rightarrow N_2 = 8.672\times10^{-9}\ (cm^{-3}) \simeq 0$

Hence $\Delta N = -N_1 = -10^{22}(cm^{-3})$, i.e. no gain but has attenuation.

$\alpha(\nu_o) = \sigma(\nu_o)(-\Delta N) = \frac{\lambda_o^2}{n^2 8\pi t_{sp}}\frac{2}{\pi\Delta\nu}N_1 = \underbrace{\frac{(0.6943\times10^{-4})^2}{1.76^2\,4\pi^2\,3\times10^{-3}}\frac{1}{330\times10^{9}}}_{3.482\times10^{-20}}$

$\times10^{22} = 398.2\,(cm^{-1})$

b) $\gamma(\nu_o) = 0.5\,cm^{-1} \Rightarrow \Delta N = \frac{\gamma(\nu_o)}{\sigma(\nu_o)} = \frac{0.5}{3.482\times10^{-20}} = 1.256\times10^{19}\,(cm^{-3})$

c) $G = e^{\gamma d} \Rightarrow d = \frac{1}{\gamma}\ln G = \frac{1}{0.5}\ln 4 = 2.773\,(cm)$

4) a) $\bar{\nu}_0 = \bar{\nu}_{E2} - \bar{\nu}_{E1} = 18340 - 2627$

$= 15713\ (cm^{-1})$ //

b) $\int_0^\infty g(\nu) d\nu = 1$ note: unit of $g(\nu)$ is in sec.

$\int_0^\infty g(\bar{\nu}) d\bar{\nu} = 1$ note: unit of $g(\bar{\nu})$ is in cm.

Area under the curve of $g(\bar{\nu}) = 1$

$\Rightarrow \frac{1}{2}\frac{k}{3} + \frac{k/3 + k}{2} \times 3 + \frac{1}{2}k = 1 \Rightarrow K = \frac{1}{\frac{1}{6} + 2 + \frac{1}{2}} = 0.375\ (cm)$

Note: $\nu = C_0 \bar{\nu} \times 10^2 \Rightarrow d\nu = C_0 \times 10^2\, d\bar{\nu}$

$\Rightarrow g(\bar{\nu}) = g(\nu) C_0 \times 10^2 \Rightarrow g(\nu) = g(\bar{\nu}) / C_0 \times 10^2$

So $k(s) = \frac{0.375}{3 \times 10^8 \times 10^2} = 1.25 \times 10^{-11}\ (s)$ //

c) $\sigma(\nu_0) = \frac{A_{21} \lambda_0^2}{8\pi n^2} g(\nu_0) = \frac{A_{21} \lambda_0^2}{8\pi n^2} K, \quad \lambda_0 = \frac{1}{\bar{\nu}_0} = 0.6364\ (\mu m)$

$n = 1, \quad A_{21} = 10\ (s^{-1})$

$\sigma(\nu_0) = \frac{10 \times (0.6364 \times 10^{-6})^2 \times 1.25 \times 10^{-11}}{8\pi} = 2.01 \times 10^{-24}\ (m^2)$

$= 2.01 \times 10^{-20}\ (cm^2)$ //

6) $\lambda_0 = 632.8\ nm, \quad A_{21} = 6.56 \times 10^6\ (s^{-1})$

a) Area of $g(\nu)$ over $\nu = \left[k\, 0.75 + 2\left(\frac{3k}{4}\right) 0.75 + 2\left(\frac{1}{2}k\right) 0.75\right] \times 10^9$

$= 1$

$\Rightarrow 3.5K\, (0.76 \times 10^9) = 1 \Rightarrow k = 3.8 \times 10^{-10}\ (s) = g(\nu_0)$

b) $\sigma = A_{21} \frac{\lambda_0^2}{8\pi} K = 6.56 \times 10^6 \frac{(0.6328 \times 10^{-6})^2}{8\pi} 3.8 \times 10^{-10}$

$= 3.972 \times 10^{-17}\ (m^2) = 3.972 \times 10^{-13}\ (cm^2)$