HW 11 Solution

1) a) Exercise 14.4-1

Ruby laser $\lambda_0 = 694.3\,nm$, Find ϕ_s and I_s at $\nu = \nu_0$, assuming $\tau_s = 2t_{sp}$. From table 14.3-1 : $2\times10^{-20}\,cm^2\,(\sigma_0)$.

$t_{sp} = 3\,ms$, $\Delta\nu = 330\,GHz$, $n = 1.76$.

$$\frac{1}{\phi_s(\nu)} = \tau_s\sigma(\nu) \Rightarrow \phi_s(\nu_0) = \frac{1}{\tau_s\sigma(\nu_0)} = \frac{1}{\tau_s\sigma_0} = \frac{1}{2\times3\times10^{-3}\times2\times10^{-20}}$$

$$= 0.833\times10^{22}\,(s^{-1}cm^{-2})$$

$$I_s = h\nu\phi_s = 1.6\times10^{-19}\,\frac{1.24}{0.6943}\times0.833\times10^{22} = 2.382\,(kW/cm^2)$$

b) Exercise 15.1-1

a) $\lambda_0 = 694.3\,nm$ $T = 300k$, $\alpha(\nu_0) = 0.2\,cm^{-1}$, $N_{total} = 1.58\times10^{19}\,cm^{-3}$

$\alpha = \sigma_0\Delta N = \sigma_0 N_{total}$ (since $N_2 \simeq 0$, $N_1 \simeq N_{total}$)

$\Rightarrow \sigma_0 = \alpha/N_{total} = 0.2/1.58\times10^{19} = 1.266\times10^{-20}\,cm^2$

b) $d = 10\,cm$, $n = 1.76$, $A = 1\,cm^2$, $\lambda_0 = 694.3\,nm$.

$R_1 = R_2 = 0.8$. This is FP cavity with gain medium filling up the full length.

$$\alpha_r = \underbrace{\cancel{\alpha_s}}_{0} + \frac{1}{2d}\ln\left(\frac{1}{R_1R_2}\right) = \frac{1}{20}\ln\left(\frac{1}{0.8^2}\right) = 2.231\times10^{-2}\,(cm^{-1})$$

c) $\sigma_0\Delta N_t = \alpha_r \Rightarrow \Delta N_t = \alpha_r/\sigma_0 = \dfrac{2.231\times10^{-2}}{1.266\times10^{-20}} = 1.762\times10^{18}\,(cm^{-3})$

2) $\Delta\nu = 3.5\,GHz$

a) $h\nu = 30783.69 - 11202.57 = 19581.12\,(cm^{-1})$

$\nu_0 = 3\times10^{10}\times19581.12 = 5.874\times10^{14}\,(Hz)$

$\lambda = \dfrac{1}{19581.12}\,(cm) = 510.696\,(nm)$ //

b) $\Delta\nu = 3.5\,GHz$

$$\Delta\lambda = \frac{\Delta\nu}{\nu_0}\lambda = \frac{3.5\times10^9}{5.874\times10^{14}}\times510.696 = 0.003043\,(nm) = 0.03043\,(\mathring{A})$$ //

c) $$\sigma(\nu_0) = g(\nu_0)\frac{\lambda^2}{8\pi n^2}A_{21} = \frac{2}{\pi\,3.5\times10^9}\,\frac{(510.696\times10^{-9})^2}{8\pi\times1}\,3.4\times10^6$$

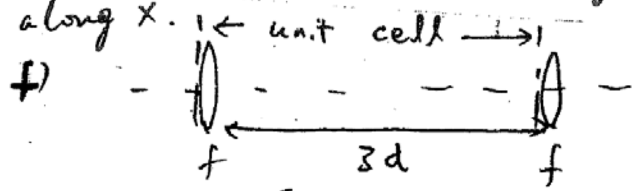
$$= 6.418\times10^{-18}\,(m^2) = 6.418\times10^{-14}\,(cm^2)$$ //

d) $d_p = \gamma_t = \left[\ln\left(\frac{1}{R_1 R_2 R_3 R_4 T_{w_1} T_{w_2}}\right)\right]\frac{1}{l_g} = \frac{1}{0.15}\ln\left(\frac{1}{0.95\times0.8\times0.96\times0.94\times0.98 \times 0.99}\right)$

$= 2.716\,(m^{-1}) = 0.02716\,(cm^{-1})$, $\Delta N_{t} = \frac{\gamma_t}{\sigma} = 4.23\times10^{11}\,(cm^{-3})$

e) Brewster angle for p-polarized light, i.e. ~~pumped~~ parallel to the plane of incident.
Since propagation is along z, polarized must be along x.

f)



ray matrix $= \begin{bmatrix} 1 & 3d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{3d}{f} & 3d \\ -\frac{1}{f} & 1 \end{bmatrix}$

$\frac{A+D}{2} = 1 - \frac{3}{2}\frac{d}{f} = 1 - \frac{3}{2}\times\frac{2}{7} = \frac{4}{7} < 1 \Rightarrow$ stable.

3) Exercise 14.2-3

a) At zero population difference, there is no saturation and stimulated emission. We consider ΔN_0

3-level system: $\Delta N_0 = 0 \Rightarrow 0 = \frac{N_{total}(t_{sp}W - 1)}{1 + t_{sp}W}$

$\Rightarrow W = \frac{1}{t_{sp}}$

4-level system: $\frac{t_{sp} N_{total} W}{1 + t_{sp}W} = 0 \Rightarrow W = 0$

b) 3-level system: $W = \frac{2}{t_{sp}}$, $\Delta N_0 = \frac{N_{total}(2-1)}{1+2} = \frac{N_{total}}{3}$

4-level system: $W = \frac{1}{2t_{sp}}$, $\Delta N_0 = \frac{N_{total}\frac{1}{2}}{1+\frac{1}{2}} = \frac{N_{total}}{3}$

3-level system: $R_{32} = \frac{1}{2}(N_{total} - \Delta N)W = \frac{1}{2}\left(\frac{2}{3}\right) N_{total}\frac{2}{t_{sp}} = \frac{2}{3}\frac{N_{total}}{t_{sp}}$

4-level system: $R_{42} = \frac{2}{3} N_{total}\frac{1}{2t_{sp}} = \frac{1}{3}\frac{N_{total}}{t_{sp}}$

Pump Power $= R \cdot h\nu_p \cdot V_{2\,vol}$. Assume pump freq ν_p is the same for both systems. Therefore, $\frac{P_{42}}{P_{32}} = \frac{R_{42}}{R_{32}} = \frac{1/3}{2/3} = \frac{1}{2}$

4-level system only require 50% of power to achieve the same population differences as that of 3-level system.