due 9/25/2017

HW 2

1. A plane wave has electric field $\vec{E} = 5\hat{y}\cos(2\pi \times 10^{10} t + 200\pi x)$. The medium has $\mu = \mu_o$. Find (a) frequency in HZ, polarization, wave vector $\vec{k}$, (b) phase velocity c and permittivity ε of the medium, (c) the corresponding magnetic field $\vec{H}$ in phasor form, (d) average Poynting vector $< \vec{S} >$. (e) power within a circle of radius = 2 meters. (20 points)

2. a) Derive Eqs. (5.5-7) and (5.5-8) for weakly absorbing media from Eq. (5.5-5) on page 172. Also find the expressions for the real part and imaginary part of η. (5 points)
   b) Derive Eqs. (5.5-11) and (5.5-12) on page 173 from Eq. (5.5-5). Also find the expressions for the real part and imaginary part of η. (5 points)

3. Exercise 5.5-1 on page 172. (10 points)

Bonus problem for undergraduate but required for graduate students

4. Derive Eqs. (5.4-3) and (5.4-4) on page 165 by substituting Eqs. (5.4-1) and (5.4-2) into Maxwell's equations. (10 points)
   Useful vector identity: $\nabla \times (f\vec{A}) = (\nabla f) \times \vec{A} + f\nabla \times \vec{A}$.

Extra-credit

5. The algebraic forms of Maxwell's equation for plane wave propagating in a linear homogeneous **anisotropic** medium are

$$\vec{k} \times \vec{H} = -\,\omega\vec{D}$$

$$\vec{k} \times \vec{E} = \omega\vec{B}$$

   where $\vec{B}$ relates to $\vec{H}$ and $\vec{D}$ relates to $\vec{E}$ by $\vec{B} = \mu_o(\vec{H} + \vec{M})$ and $\vec{D} = \varepsilon_o\vec{E} + \vec{P}$
   For many materials, the polarization vector $\vec{P}$ is not collinear with $\vec{E}$; hence, $\vec{D}$ is not collinear with $\vec{E}$ either. The same comments apply to $\vec{B}$ and $\vec{M}$ and $\vec{H}$. Assume a dielectric medium with $\vec{M} = 0$ (non-magnetic medium) but with no restrictions placed on $\vec{D}$ and $\vec{E}$.
   a) Show that $\vec{k} \cdot \vec{D} = 0$.
   b) Show that the wave vector $\vec{k}$ always points in the direction of $\vec{D} \times \vec{B}$.
   c) Show that the amplitude of the wave vector $\vec{k}$ is given by

$$|\vec{k}|^2 = \omega^2 \mu_o \frac{\vec{D} \cdot \vec{D}}{\vec{E} \cdot \vec{D}}$$

   d) Show that the Poynting vector, $\vec{S} = \vec{E} \times \vec{H}^*/2$, can point in a direction other than that of the wave vector $\vec{k}$.