HW 3

1) a) (5.5-19) $$\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2-\nu^2+j\nu\Delta\nu} \cdot \frac{(\nu_0^2-\nu^2)-j\nu\Delta\nu}{(\nu_0^2-\nu^2)-j\nu\Delta\nu}$$

$$= \chi_0 \frac{\nu_0^2(\nu_0^2-\nu^2)-j\nu_0^2\nu\Delta\nu}{(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2}$$

(5.5-20) $$\chi'(\nu) = Re(\chi(\nu)) = \chi_0 \frac{\nu_0^2(\nu_0^2-\nu^2)}{(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2}$$

(5.5-21) $$\chi''(\nu) = Im(\chi(\nu)) = -\chi_0 \frac{\nu_0^2\nu\Delta\nu}{(\nu_0^2-\nu^2)^2+(\Delta\nu\,\nu)^2}$$

Use approx, $\nu \simeq \nu_0$ $\quad \nu_0^2-\nu^2 = (\nu_0-\nu)(\nu_0+\nu) \simeq (\nu_0-\nu)2\nu_0$

(5.5-23) $$\chi''(\nu) \simeq -\chi_0 \frac{\nu_0^2\,\nu_0\,\Delta\nu}{4\nu_0^2(\nu_0-\nu)^2+\nu_0^2(\Delta\nu)^2} = -\chi_0 \frac{\nu_0\Delta\nu}{4}\,\frac{1}{(\nu_0-\nu)^2+(\Delta\nu/2)^2}$$

$$\chi'(\nu) \simeq \chi_0 \frac{\nu_0^2(\nu_0-\nu)\,2\nu_0}{4\nu_0^2(\nu_0-\nu)^2+\nu_0^2(\Delta\nu)^2} = \chi_0 \frac{(\nu_0-\nu)\nu_0}{2}\,\frac{1}{(\nu_0-\nu)^2+(\Delta\nu/2)^2}$$

$$= -\chi''(\nu)\,\frac{2(\nu_0-\nu)}{\Delta\nu} = \chi''(\nu)\,\frac{2(\nu-\nu_0)}{\Delta\nu} \quad (5.5\text{-}24)$$

b) At $\nu_0 = 0$ $\quad g(\nu) = \dfrac{\Delta\nu}{2\pi(\nu^2+(\Delta\nu/2)^2)}$. Max at $\nu = 0$

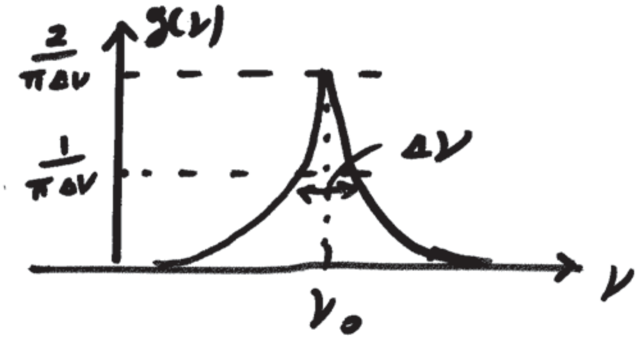
$g(0) = \dfrac{2}{\pi\Delta\nu} = g_{max}$. Now find ν so that $g(\nu) = \frac{1}{2}g(0)$

$$\Rightarrow \frac{1}{\pi\Delta\nu} = \frac{\Delta\nu}{2\pi(\nu^2+(\Delta\nu/2)^2)} \Rightarrow 2 = \frac{\Delta\nu^2}{\nu^2+(\Delta\nu/2)^2}$$

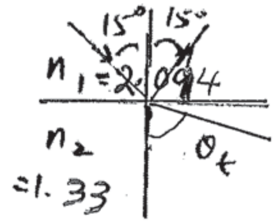
$$\Rightarrow \frac{1}{2} = \frac{(\Delta\nu/2)^2}{\nu^2+(\Delta\nu/2)^2} \Rightarrow \nu = \pm\Delta\nu/2$$ Let $\nu_+ = \Delta\nu/2$

and $\nu_- = -\Delta\nu/2$, bandwidth $= \nu_+ - \nu_- = \Delta\nu$ (FWHM)
(Full width)

$\lim_{\nu\to\infty} g(\nu) = 0$



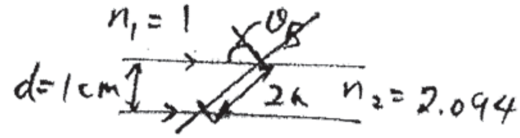
2) 1.16 (c) $n_1 \sin 15° = n_2 \sin \theta_t$



$$\theta_t = \sin^{-1}\left(\frac{2.094}{1.33} \sin 15°\right)$$

$$= 24.05°$$

1.18 (b) $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$



$$2a = \frac{d}{\cos\theta_B} = \frac{d\sqrt{n_1^2 + n_2^2}}{n_1} = 2.321\ (\text{cm})$$

3) a) $r_\parallel = \dfrac{-n_2 \cos\theta_i + n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$, $r_\perp = \dfrac{-n_2 \cos\theta_t + n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_i}$

$r_\parallel = r_\perp$ if $\theta_i = \theta_t$. Since $n_1 \neq n_2$, it is only possible for normal incident, i.e. $\theta_i = \theta_t = 0$. Now $R = |r|^2$

$$\Rightarrow R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

b) 1.19 (a)

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \left(\frac{1.4679 - 2.41}{1.4679 + 2.41}\right)^2$$

$\xrightarrow{}$ $n_1 = 2.41$ | $\xrightarrow{}$ $n_2 = 1.4679$

$$= 0.059$$

4) From the September 19 summary,

a) $n_1(E_{oi} - E_{or}) = n_2 E_{ot}$ ①

$E_{oi} + E_{or} = E_{ot}\dfrac{\cos\theta_t}{\cos\theta_i}$ ②

①/②: $\dfrac{E_{oi}-E_{or}}{E_{oi}+E_{or}} = \dfrac{n_2\cos\theta_i}{n_1\cos\theta_t} \Rightarrow \dfrac{E_{oi}+E_{or}-(E_{oi}-E_{or})}{E_{oi}-E_{or}+(E_{oi}+E_{or})} = \dfrac{n_1\cos\theta_t - n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t}$

$\Rightarrow \dfrac{E_{or}}{E_{oi}} = \dfrac{n_1\cos\theta_t - n_2\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} = r_{\parallel}$ — ③

b) $n_1\sin\theta_i = n_2\sin\theta_t \Rightarrow n_2 = n_1\sin\theta_i/\sin\theta_t$ ④

Substitute ④ into ③:

$$r_{\parallel} = \frac{n_1\cos\theta_t\sin\theta_t - n_1\sin\theta_i\cos\theta_i}{n_1\cos\theta_i\sin\theta_i + n_1\cos\theta_t\sin\theta_t} = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_i + \sin 2\theta_t}$$

$$= \frac{2\sin(\theta_t-\theta_i)\cos(\theta_t+\theta_i)}{2\sin(\theta_t+\theta_i)\sin(\theta_t-\theta_i)} = \frac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}$$

substitute ④ into $r_{\perp} = \dfrac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = \dfrac{n_1\cos\theta_i\sin\theta_t - n_1\cos\theta_t\sin\theta_i}{n_1\cos\theta_i\sin\theta_t + n_1\cos\theta_t\sin\theta_i}$

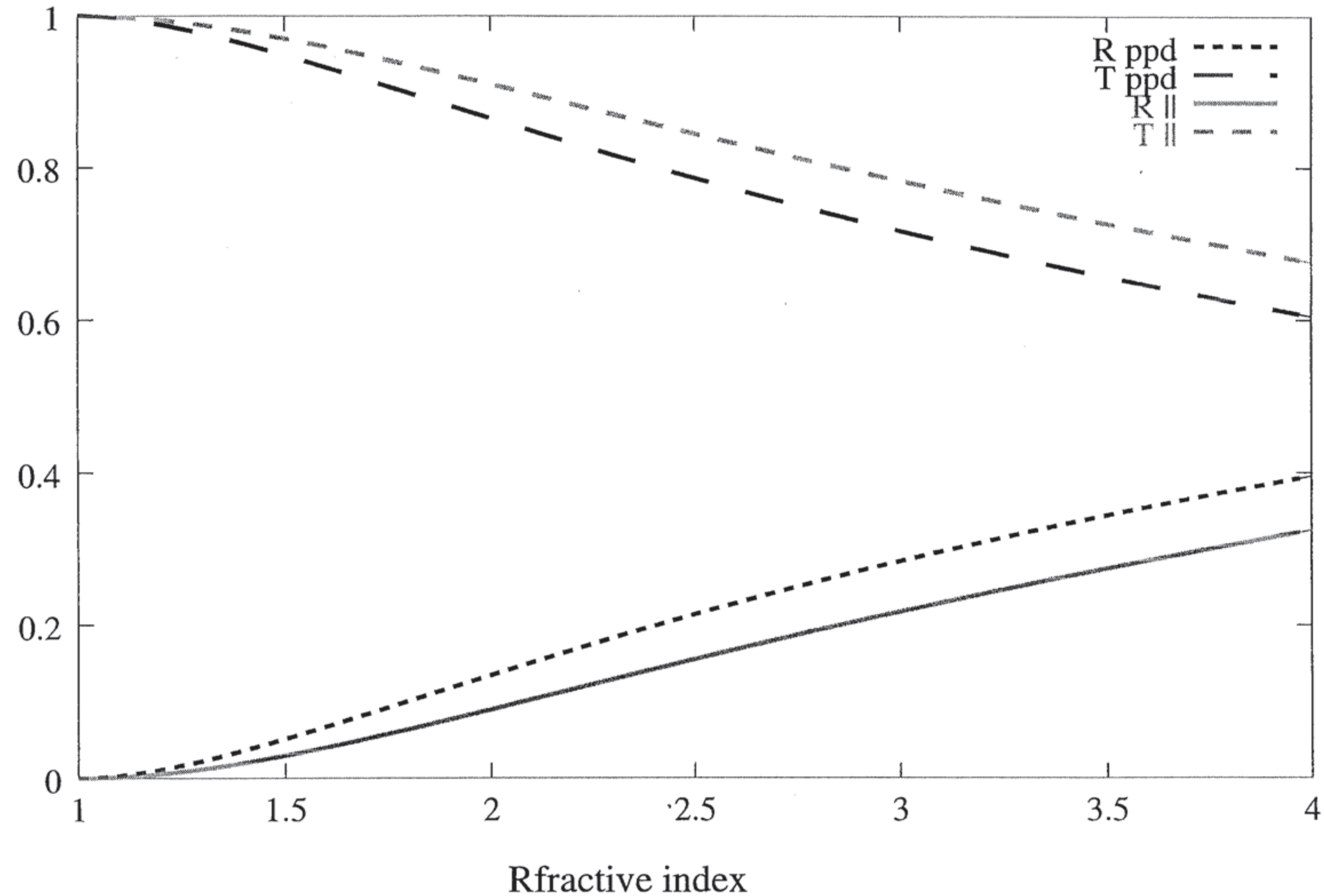
$$r_{\perp} = \frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}$$

5) 1.22 $\theta_{in} = 25°$, n_2: 1 to 4, $n_1 = 1$

Plot $R_{\parallel}$ & $R_{\perp}$ & $T_{\parallel}$ & $T_{\perp}$ versus n_2.

$$R_{\perp} = \frac{\sin^2(\theta_i-\theta_t)}{\sin^2(\theta_i+\theta_t)} = \frac{\sin^2\left(25° - \sin^{-1}\left(\frac{\sin 25°}{n_2}\right)\right)}{\sin^2\left(25° + \sin^{-1}\left(\frac{\sin 25°}{n_2}\right)\right)}, \quad T_{\perp} = 1 - R_{\perp}$$

$$R_{\parallel} = \frac{\tan^2(\theta_i-\theta_t)}{\tan^2(\theta_i+\theta_t)} = \frac{\tan^2\left(25° - \sin^{-1}\left(\frac{\sin 25°}{n_2}\right)\right)}{\tan^2\left(25° + \sin^{-1}\left(\frac{\sin 25°}{n_2}\right)\right)}, \quad T_{\parallel} = 1 - R_{\parallel}$$



gnuplot script for problem 1.22

```
set terminal fig color big
set output 'hw2.fig'
plot [t=1:4] (sin(0.43633-asin(sin(0.43633)/t))/sin(0.43633+asin(sin(0.43633)/t)))**2 titl
e "R ppd" with lines, \
1 - (sin(0.43633-asin(sin(0.43633)/t))/sin(0.43633+asin(sin(0.43633)/t)))**2 title "T ppd"
 with lines, \
(tan(0.43633-asin(sin(0.43633)/t))/tan(0.43633+asin(sin(0.43633)/t)))**2 title "R ||" with
 lines, \
1-(tan(0.43633-asin(sin(0.43633)/t))/tan(0.43633+asin(sin(0.43633)/t)))**2 title "T ||" wi
th lines
```