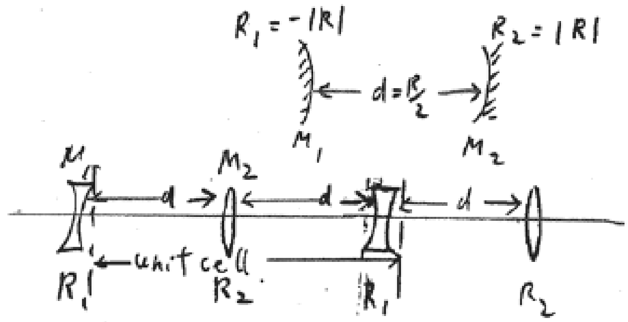
HW 5 Solution

1) $\frac{A+D}{2} = 1 - \frac{3d}{2f}$

$-1 \le 1 - \frac{3d}{2f} \le 1 \Rightarrow 0 \le \frac{1}{2}\left(2 - \frac{3d}{2f}\right) \le 1 \Rightarrow 0 \le 1 - \frac{3d}{4f} \le 1$ or $0 \le \frac{d}{f} \le \frac{4}{3}$

2) a), b)



c) $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_T = \begin{bmatrix} 1 & d \\ -\frac{1}{f_1} & \left(1-\frac{d}{f_1}\right) \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{1}{f_2} & \left(1-\frac{d}{f_2}\right) \end{bmatrix}$

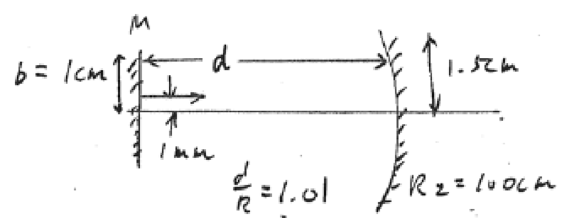
$= \begin{bmatrix} 1 - \frac{d}{f_2} & d + d\left(1-\frac{d}{f_2}\right) \\ -\frac{1}{f_1} - \frac{1}{f_2}\left(1-\frac{d}{f_1}\right) & \left(1-\frac{d}{f_1}\right)\left(1-\frac{d}{f_2}\right) - \frac{d}{f_1} \end{bmatrix}$ $\left(d = \frac{R}{2},\ f_2 = \frac{R}{2},\ f_1 = -\frac{R}{2} \right)$

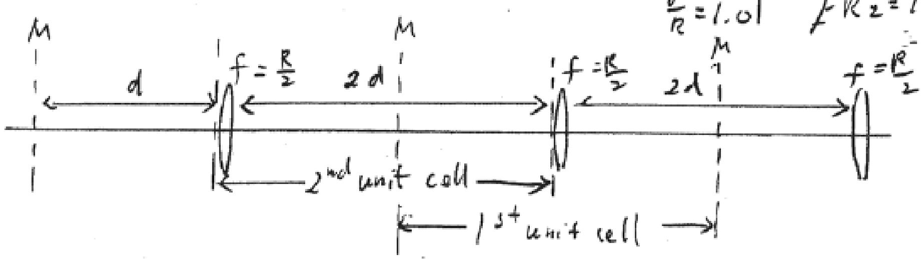
$= \begin{bmatrix} 0 & d \\ -\frac{2}{R} & 1 \end{bmatrix}$

d) stable if $0 \le \frac{A+D+2}{4} \le 1$ and $\frac{A+D+2}{4} = \frac{3}{4}$ in this case

So it is stable!

3) a)





b) T-matrix for 1st unit cell:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{bmatrix} = \begin{bmatrix} 1-\frac{d}{f} & 2d\left(-\frac{d}{2f}\right) \\ -\frac{1}{f} & 1-\frac{d}{f} \end{bmatrix}$$

i) $F_1 = \frac{A+D}{2} + \left[\left(\frac{A+D}{2}\right)^2 - 1\right]^{\frac{1}{2}}$ (Note: $\frac{A+D}{2} = 1 - \frac{d}{f} = -1.02$)

$= -1.02 + [(1.02)^2 - 1]^{\frac{1}{2}} = -0.819$ //

$F_2 = \frac{A+D}{2} - \left[\left(\frac{A+D}{2}\right)^2 - 1\right]^{\frac{1}{2}} = -1.221$ //

ii) $r_0 = r_a + r_b = 1$ mm

$r_1 = r_0 A + r_0' B = r_a F_1 + r_b F_1$ and $r_0' = 0$

$r_b = \frac{1}{F_1 - F_2}[r_0(F_1 - A) - B r_0'] = \frac{1}{F_1 - F_2}(F_1 - A)$

$= 0.5$ (mm) //

$r_a = \frac{1}{F_2 - F_1}[r_0(F_2 - A) - B r_0'] = \frac{1}{F_2 - F_1}(F_2 - A) = 0.5$ (mm) //

c) Since $|F_2| > |F_1|$, F_2 dominates, i.e. $M \simeq |r_b F_2^m|$

Estimate outside the flat mirror when

$b \simeq r_b (1.221)^m$

$\Rightarrow m \simeq \log\left(\frac{1}{0.05}\right) / \log(1.221) \simeq 15.003$

Since m is integer, we check whether m=15 satisfies the condition.

$r_{15} = 0.5\,(-0.819)^{15} + 0.5\,(-1.221)^{15} = 10.02$ (mm)

$\therefore r_{15} > b \Rightarrow$ m=15 does satisfy the condition

Notice that m is for round trip and a round trip has 2 passes, i.e. 30 passes.

4) $L = 25$ cm, $\lambda = 632.8$ nm, $n = 1$

$2Ln = p\,\lambda_p$ —— ①

To find the closest mode to λ, we set $\lambda_p = \lambda$ and find p

$p = \mathrm{int}\left[\frac{2 \times 25 \times 10^{-2}}{632.8 \times 10^{-9}}\right] = 790139$ (closest integer)

λ_p = Actual lasing wavelength = $\frac{2nL}{p} = 632.80005$ (nm)

$$\nu = \frac{c_0}{n\lambda_p} = 4.73755\times10^{14} (Hz) = p\,\Delta\nu_{FSR}$$

$$\Delta L = \left(\frac{1}{L}\frac{dl}{dT}\right)\Delta T\cdot L = 5.5\times10^{-7}\times25\times25\times10^{-2} = 3.4375\times10^{-6} (m)$$

From ①, we know $2\cdot\Delta L\cdot n = p\,\Delta\lambda_p$ or $\frac{|\Delta L|}{L} = \frac{|\Delta\lambda|}{\lambda_p}$

$$\Rightarrow \Delta\lambda_p = \frac{2\cdot\Delta L\cdot n}{p} = \frac{2\times3.4375\times10^{-6}}{790139} = 8.701\times10^{-12}$$

And $\frac{\Delta\nu}{\nu} = \frac{\Delta\lambda_p}{\lambda_p} \Rightarrow \Delta\nu = \nu\frac{\Delta\lambda_p}{\lambda_p} = p\,\Delta\nu_{FSR}\,\frac{8.701\times10^{-12}}{632.80005\times10^{-9}}$

$$\Delta\nu = 10.86\,\Delta\nu_{FSR}$$

6) d) ABCD matrix for 2nd unit cell

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 2d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{2d}{f} & 2d \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$$F_1 = \frac{A+D}{2} + \left[\left(\frac{A+D}{2}\right)^2 - 1\right]^{\frac{1}{2}} \qquad F_2 = \frac{A+D}{2} - \left[\left(\frac{A+D}{2}\right)^2 - 1\right]^{\frac{1}{2}}$$

Since $\frac{A+D}{2}$ is constant for a specific system, $F_{1,2}$ are the same as before, i.e. $F_1 = -0.819$, $F_2 = -1.221$

$$r_b = \frac{1}{F_1 - F_2}(F_1 - A) = 5.525 (mm)$$

$$r_a = \frac{1}{F_2 - F_1}(F_2 - A) = -4.525 (mm)$$

Estimate Outside the converging mirror when

$$15 \simeq |5.525\,(1.221)^m|$$

$$m \simeq \log\left(\frac{15}{5.525}\right)/\log 1.221 \simeq 5.002$$

Actually, $r_5 = -4.525\,(-0.819)^5 + 5.525\,(-1.221)^5 = -13.3$ (mm)

$\therefore r_5$ fails

$r_6 = -4.525(-0.819)^6 + 5.525(-1.221)^6 > 15$ (mm)

$\therefore$ **m** $= \underline{\underline{6}}$ with satisfy the condition.

As a result, the ray will <u>leave</u> the cavity <u>at the</u> <u>converging ~~mirror~~</u>. And the ray will miss the spherical mirror after <u>6 and a half round trip (i.e. 13 passes)</u> because the ray has travelled a distance d from the flat mirror to the spherical mirror before the ray reaches r_0 of the 2^{nd} unit cell.

e) Power out $= 1 \times 10^{-6} \times G^{13} = 1221$ (W)

Extra see plot

7) Observe that $\theta_t = 0.012$ rad causes the mode to move more than $\Delta\nu_{FSR}$ (b) not required. Just extra info for you.

b) Require to move ν by one $\Delta\nu_{FSR}$ when $\theta_t = 0$, i.e. cause p to increase or decrease by 1, mode hopping.

$$\frac{c_0 p}{2 n_g L} = \frac{c_0 (p \pm 1)}{2 n_g L \cos\theta_t} \Rightarrow \frac{p}{p \pm 1} = \frac{1}{\cos\theta_t}$$

Solution exists when p decreases, i.e.

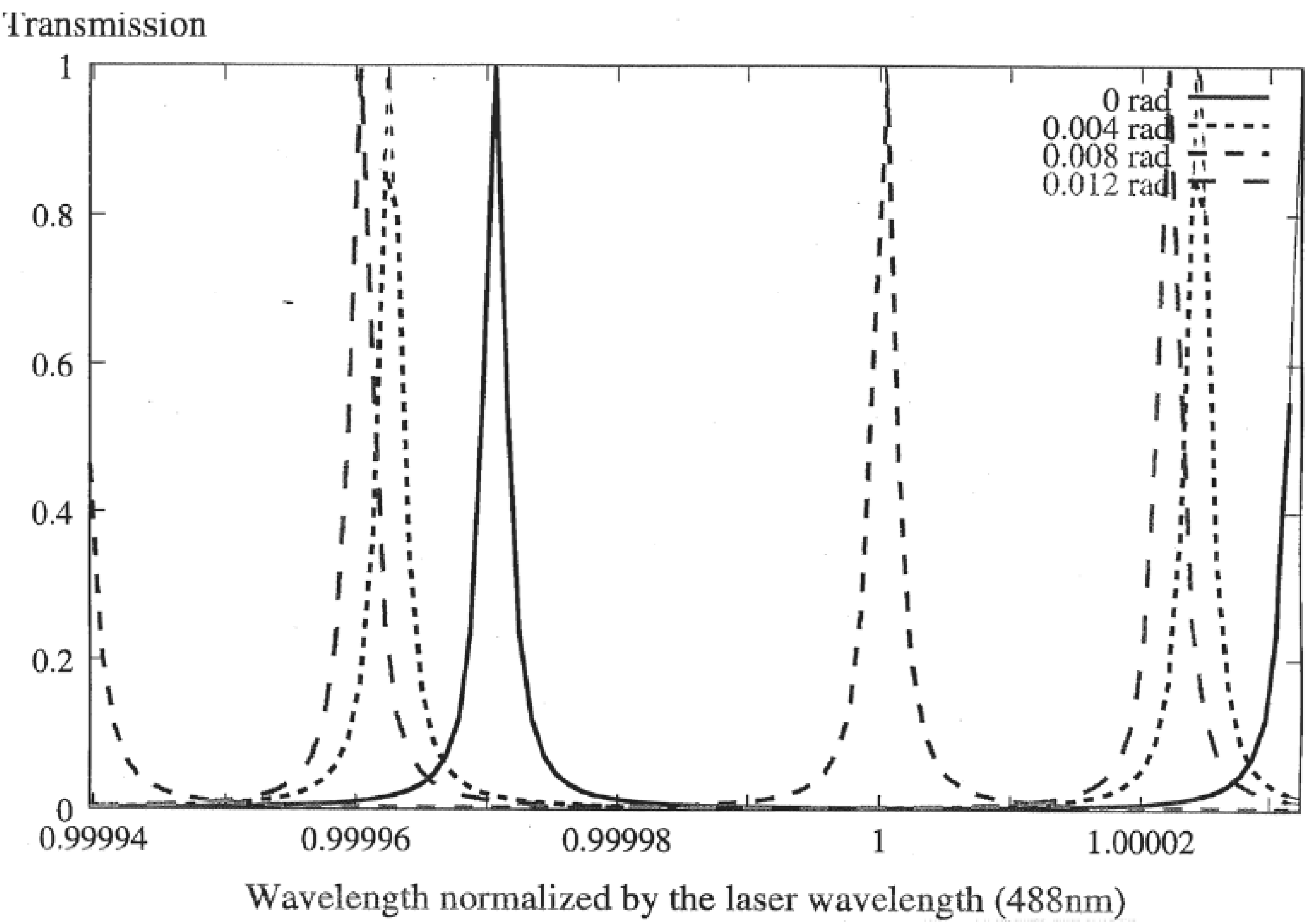
$$\frac{p}{p-1} = \frac{1}{\cos\theta_t} \Rightarrow \theta_t = \cos^{-1}\left(\frac{p-1}{p}\right)$$

$$2nL = p\lambda \Rightarrow p = \text{int}\left(\frac{2nL}{\lambda}\right) = \text{int}\left(\frac{2 \times 1.58 \times 2.5 \times 10^{-3}}{488 \times 10^{-9}}\right)$$

$$= 16189$$

$$\theta_t = \cos^{-1}\left(\frac{16188}{16189}\right) = 0.63684^\circ = 0.011 \text{ rad}$$

This answer is close to the value shown by the plot.




```
# gnuplot script for hw5 extra credit problem
set terminal fig color big
set output 'hw5ex.fig'
set samples 500
l=2.5e-3
lamb=488.e-9
n=1.58
R=0.9
T_sq=(1.-R)**2
theta1=0
theta2=0.004
theta3=0.008
theta4=0.012
pi=atan(1.)*4.
d1=4.*pi*l*n
d2=d1*cos(theta2)
d3=d1*cos(theta3)
d4=d1*cos(theta4)
p=int(2*n*l/lamb+0.5)
xmin=2*n*l/(p+0.5)/lamb
xmax=2*n*l/(p-1)/lamb
plot [lm=xmin:xmax] [0:1] T_sq /(T_sq+4*R*(sin(d1*0.5/lm/lamb))**2) title "0 rad", \
T_sq/ (T_sq+4*R*(sin(d2*0.5/lm/lamb))**2) title "0.004 rad", \
T_sq/ (T_sq+4*R*(sin(d3*0.5/lm/lamb))**2) title "0.008 rad", \
T_sq/ (T_sq+4*R*(sin(d4*0.5/lm/lamb))**2) title "0.012 rad"
```