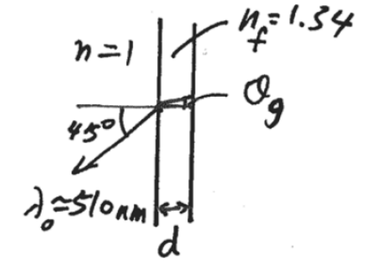
HW 6 Solution

1) $n \sin 45° = n_f \sin \theta_g$

$\theta_g = 31.85°$



At θ_g, green light is enhanced.

So resonant condition:

$$\nu = p \frac{c_0}{2 n_f d \cos\theta_g}$$

$$\Rightarrow d = \frac{c_0 p}{2 n_f \nu \cos\theta_g} = \frac{\lambda_0 p}{2 n_f \cos\theta_g} = 2.2403 \times 10^{-7} p \ (m)$$

Choose $p = 10^4 \Rightarrow d = 2.2403$ (mm)

There are many thicknesses can satisfy the resonant condition.

2) $\Delta\nu_{FSR} = \nu_F = 150$ MHz, $n = 1$

$\delta\nu = 5$ MHz

$$\Delta\nu_{FSR} = \frac{c_0}{2nd} \Rightarrow d = \frac{c_0}{2n\Delta\nu_{FSR}} = \frac{3\times10^8}{2\times150\times10^6} = 1 \ (m)$$

$\mathcal{F} = \Delta\nu_{FSR}/\delta\nu = 150/5 = 30$

There are 2 ways to find $\mathcal{R}$. $(|r| = \sqrt{\mathcal{R}})$

More exact way: $\mathcal{F} = \frac{\pi(\mathcal{R}_1\mathcal{R}_2)^{1/4}}{1-\sqrt{\mathcal{R}_1\mathcal{R}_2}} = \frac{\pi\sqrt{\mathcal{R}}}{1-\mathcal{R}} = \frac{\pi|r|}{1-|r|^2}$

$$1-|r|^2 = \frac{\pi}{\mathcal{F}}|r| \Rightarrow |r|^2 + \frac{\pi}{\mathcal{F}}|r| - 1 = 0$$

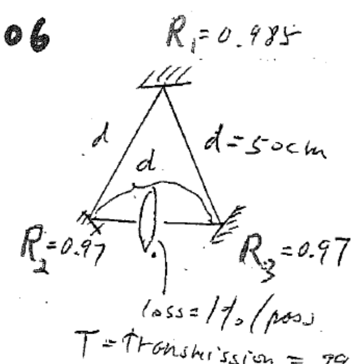
$$|r| = \left[-\frac{\pi}{\mathcal{F}} \pm \sqrt{\left(\frac{\pi}{\mathcal{F}}\right)^2+4}\right]\frac{1}{2} = -\frac{\pi}{2\mathcal{F}} \pm \sqrt{\left(\frac{\pi}{2\mathcal{F}}\right)^2+1} = 0.949$$ (choose positive solution)

$R = |r|^2 = 0.9006$

Approx method: $\alpha_r = \frac{1}{2d}\ln\left(\frac{1}{\mathcal{R}_1\mathcal{R}_2}\right) = \frac{1}{d}\ln\left(\frac{1}{\mathcal{R}}\right)$

$$\mathcal{F} \simeq \frac{\pi}{\alpha_r d} = \frac{\pi}{\ln\left(\frac{1}{\mathcal{R}}\right)} \Rightarrow \mathcal{R} \cong e^{-\pi/\mathcal{F}} = 0.9006$$

3)



a) $\Delta\nu_{FSR} = \frac{c_0}{3nd} = \frac{3\times10^8}{3\times1\times50\times10^{-2}} = 2\times10^8$ (Hz)

b) $\tau_p = \frac{1}{\Delta\nu_{FSR}F_l}$

round trip loss $F_l = 1 - R_1 R_2 R_3 (T)^2 = 1 - 0.985\times0.97\times0.97 \times(1-0.01)^2$

$$F_\ell = 0.09166$$

$$\tau_p = \frac{1}{2\times10^8 \times 0.09166} = 54.55\,(ns)$$

c) $\mathcal{F} = \frac{\Delta\nu_{FSR}}{\delta\nu}$ and $\delta\nu = \frac{1}{2\pi\,\tau_p} = 2.918\times10^6\,(Hz)$

$$\mathcal{F} = \frac{2\times10^8}{2.918\times10^6} = 68.55$$

d) $Q = \frac{\nu_o}{\delta\nu} = \frac{c_o}{\delta\nu}\frac{1}{\lambda_o} = \frac{3\times10^8}{2.918\times10^6}\frac{1}{0.6428\times10^{-6}} = 1.6\times10^8$

e) $\frac{\delta\lambda}{\lambda_o} = \frac{\delta\nu}{\nu_o} \Rightarrow \delta\lambda = \frac{\lambda_o^2}{c_o}\delta\nu = \frac{(0.6428\times10^{-6})^2}{3\times10^8}2.918\times10^6 = 4.019\times10^{-6}\,(nm)$

Extra Credit

a) We know $|\Delta\nu|/\nu = |\Delta\lambda|/\lambda = |\Delta L|/L$

So $\boxed{\lambda = \left|\frac{\Delta\lambda}{\Delta L}\right| L}$. Now $\Delta\nu = \Delta\nu_{FSR} \Rightarrow \Delta\lambda = \Delta\lambda_{FSR} = \frac{\Delta\nu_{FSR}}{\nu}\lambda = \frac{\lambda}{p}$

$$\lambda = \frac{\lambda/p}{\Delta L_{FSR}} L \Rightarrow p = \frac{L}{\Delta L_{FSR}}$$

$$\lambda = \frac{2nL}{p} = \frac{2nL}{L/\Delta L_{FSR}} \Rightarrow \boxed{\lambda = 2n\,\Delta L_{FSR}}$$

From the diagram 6.7cm corresponds to 0.4μm in ΔL
5.5cm " " 0.3283μm in ΔL_{FSR}

$$\lambda = 2\times0.3283\,\mu m = 0.6566\,\mu m$$

b) $\mathcal{F} = \frac{\Delta\nu_{FSR}}{\delta\nu} = \frac{\Delta L_{FSR}}{\delta L} = \frac{5.5}{0.9} = 6.11$

c) $\frac{\Delta\nu}{\nu} = \frac{\Delta L}{L} \Rightarrow \delta\nu = \frac{\delta L}{L}\nu$ and $\delta L = \frac{0.9}{6.7}\times0.4 = 0.05373\,(\mu m)$

$$\delta\nu = \frac{0.05373}{0.02}\times\frac{3\times10^8}{0.6566} = 1.23\,(GHz)$$

d) $Q = \frac{\nu}{\delta\nu} = \frac{L}{\delta L} = \frac{0.02}{0.05373\times10^{-6}} = 3.72\times10^5$

e) $\tau_p = \frac{1}{2\pi\,\delta\nu} = \frac{1}{2\pi\times1.23\times10^9} = 0.1294\,(ns)$