From Keren, “Optical Fiber Communications,” again to Sec. 3.4.3 and 4.4.5 at end.

where \( P_{\text{p, in}} \) is the input pump power, and \( \lambda_p \) and \( \lambda_s \) are the pump and signal wavelengths, respectively. The fundamental physical principle here is that the amount of signal energy that can be extracted from an EDFA cannot exceed the pump energy that is stored in the device. The inequality in Eq. (11-16) reflects the possibility of effects such as pump photons being lost due to various causes (such as interactions with impurities) or pump energy lost due to spontaneous emission.

From Eq. (11-16), we see that the maximum output signal power depends on the ratio \( \lambda_p / \lambda_s \). For the pumping scheme to work, we need to have \( \lambda_p < \lambda_s \), and, to have an appropriate gain, it is necessary that \( P_{\text{s, in}} \ll P_{\text{p, in}} \). Thus, the power conversion efficiency (PCE), defined as

\[
PCE = \frac{P_{\text{s, out}}}{P_{\text{s, in}}} = \frac{P_{\text{s, out}}}{P_{\text{p, in}}} \leq \frac{\lambda_p}{\lambda_s} \leq 1
\]  

is less than unity. The maximum theoretical value of the PCE is \( \lambda_p / \lambda_s \). For absolute reference purposes, it is helpful to use the quantum conversion efficiency (QCE), which is wavelength-independent and is defined by

\[
QCE = \frac{\lambda_s}{\lambda_p} \times PCE
\]  

The maximum value of QCE is unity, in which case all the pump photons are converted to signal photons.

We can also rewrite Eq. (11-16) in terms of the amplifier gain \( G \). Assuming there is no spontaneous emission, then

\[
G = \frac{P_{\text{s, out}}}{P_{\text{s, in}}} \leq 1 + \frac{\lambda_p P_{\text{p, in}}}{\lambda_s P_{\text{s, in}}}
\]  

This shows an important relationship between signal input power and gain. When the input signal power is very large so that \( P_{\text{s, in}} \gg (\lambda_p / \lambda_s) P_{\text{p, in}} \), then the maximum amplifier gain is unity. This means that the device is transparent to the signal. From Eq. (11-19), we also see that in order to achieve a specific maximum gain \( G \), the input signal power cannot exceed a value given by

\[
P_{\text{s, in}} \leq \frac{(\lambda_p / \lambda_s) P_{\text{p, in}}}{G - 1}
\]  

**Example 11-3.** Consider an EDFA being pumped at 980 nm with a 30-mW pump power. If the gain at 1550 nm is 20 dB, then, from Eq. (11-20), the maximum input power is

\[
P_{\text{s, in}} \leq \frac{980 / (1550 / 30 \text{ mW})}{100 - 1} = 190 \mu\text{W}
\]
From Eq. (11-16), the maximum output power is
\[ P_{\text{out}}(\text{max}) = P_{\text{in}}(\text{max}) + \frac{1}{\lambda_0} P_{\text{p,in}} = 190 \mu W + 0.63(30 \text{ mW}) = 19.1 \text{ mW} = 12.8 \text{ dBm} \]

In addition to pump power, the gain also depends on the fiber length. The maximum gain in a three-level laser medium of length \( L \), such as an EDFA, is given by
\[ G_{\text{max}} = \exp(\rho \sigma_L L) \]  
(11-21)
where \( \sigma_L \) is the signal-emission cross section and \( \rho \) is the rare-earth element concentration. When determining the maximum gain, Eqs. (11-19) and (11-21) must be considered together. Consequently, the maximum possible EDFA gain is given by the lowest of the two gain expressions:
\[ G \leq \min \left\{ \exp(\rho \sigma_L L), 1 + \frac{\lambda_0}{\lambda_2} \frac{P_{\text{p,in}}}{P_{\text{in}}} \right\} \]  
(11-22)

Since \( G = P_{\text{out}}/P_{\text{in}} \), it follows similarly that the maximum possible EDFA output power is given by the minimum of the two expressions:
\[ P_{\text{out}} \leq \min \left\{ P_{\text{in}}, \exp(\rho \sigma_L L), P_{\text{in}} + \frac{\lambda_0}{\lambda_2} P_{\text{p,in}} \right\} \]  
(11-23)

Figure 11-6 illustrates the onset of gain saturation for various doped-fiber lengths as the pumping power increases.\(^{23}\) As the fiber length increases for low pumping powers, the gain starts to decrease after a certain length because the pump does not have enough energy to create a complete population inversion in the downward portion of the amplifier. In this case, the unpumped region of the fiber absorbs the signal, thus resulting in signal loss rather than gain in that section.

Since the metastable level in an EDFA has a relatively long lifetime, it is possible to obtain very high saturated output powers. The saturated output power (the power at which gain saturation occurs) is defined as the 3-dB compression point of the small-signal gain.\(^{24}\) For large signal operation, the saturated gain increases linearly with pump power, as can be inferred from Fig. 11-7. This figure shows that as the input power increases for a given pump level, the amplifier gain remains constant until saturation occurs.

- **Example 11-4.** From Fig. 11-6 we see that for 1480-nm pumping, a 35-dB gain can be achieved with a pump power of 5 mW for an amplifier length of 30 m.

### 11.4 Amplifier Noise

The dominant noise generated in an optical amplifier is amplified spontaneous emission (ASE). The origin of this is the spontaneous recombination of electrons and holes in the amplifier medium (transition 5 in Fig. 11-4). This recombination gives rise to a broad spectral background of photons that get amplified along with the optical signal. This effect is shown in Fig. 11-8 for an EDFA amplifying a
signal at 1540 nm. The spontaneous noise can be modeled as a stream of random infinitely short pulses that are distributed all along the amplifying medium. Such a random process is characterized by a noise power spectrum that is flat with frequency. The power spectral density of the ASE noise is

$$S_{ASE}(f) = h
\nu_{eq}(G_0(f) - 1) = P_{ASE}/\Delta\nu_{opt}$$

where $P_{ASE}$ is the ASE noise power in an optical bandwidth $\Delta\nu_{opt}$ and $n_{sp}$ is the spontaneous-emission or population-inversion factor is defined as

$$n_{sp} = \frac{n_2}{n_2 - n_1}$$

where $n_1$ and $n_2$ are the fractional densities or populations of electrons in states 1, and 2, respectively. Thus, $n_{sp}$ denotes how complete the population inversion is between two energy levels. From Eq. (11-25) $n_{sp} \geq 1$, with equality holding for an ideal amplifier when the population inversion is complete. Typical values range from 1.4 to 4, depending on the wavelength and the pumping rate.

The ASE noise level depends on whether codirectional or counterdirectional pumping is used. Figure 11-9 shows experimental and calculated data on ASE noise versus pump power for different EDFA lengths.

Since ASE originates ahead of the photodiode, it gives rise to three different noise components in an optical receiver in addition to the normal thermal noise of the photodetector. This occurs because the photodetector consists of a number of beat signals between the signal and the optical noise fields, in addition to the squares of the signal field and the spontaneous-emission field. If the total optical field is the sum of the signal field $E_s$ and the spontaneous-emission field $E_n$, then the total photodetector current $i_{det}$ is proportional to the square of the electric field of the optical signal: $i_{det} \propto (E_s + E_n)^2 = E_s^2 + E_n^2 + 2E_s \cdot E_n$. Here the first two terms arise purely from the signal and noise, respectively. The third term is a mixing component (a beat signal) between the signal and noise, which can fall within the bandwidth of the receiver and degrade the signal-to-noise ratio. First, taking the ASE photons into account, the optical power incident on the photodetector becomes $P_0 = GP_{in} + S_{ASE} \Delta \nu_{opt}$. Note that $\Delta \nu_{opt}$ can be reduced significantly if an optical filter precedes the photodetector. Substituting this expression for $P_0$ into Eq. (6-6) then yields the total mean-square shot-noise current

$$i_{shot}^2 = \sigma_{shot}^2 = \sigma_{shot+}^2 + \sigma_{shot-ASE}^2 = 2qG \rho P_{in}B + 2qS_{ASE} \Delta \nu_{opt}B$$

where $B$ is the front-end receiver electrical bandwidth.

The other two arises from the mixing of the different optical frequencies contained in the light signal and the ASE, which generates two sets of beat frequencies. Since the signal and the ASE have different optical frequencies, the beat noise of the signal with the ASE is

$$\sigma_{ASE}^2 = 4(\rho G \rho P_{in})(\rho S_{ASE}B)$$

In addition, since the ASE spans a wide optical frequency range, it can beat against itself giving rise to the noise current (see Sec. 9.3.1)

$$\sigma_{ASE-ASE}^2 = A \Delta S_{ASE}^2 (2\Delta \nu_{opt} - B)B$$

The total mean-square receiver noise current then becomes

$$i_{total}^2 = \sigma_{total}^2 = \sigma_1^2 + \sigma_{shot+}^2 + \sigma_{shot-ASE}^2 + \sigma_{ASE}^2 + \sigma_{ASE-ASE}^2$$

where the thermal noise variance $\sigma_1^2$ is given by Eq. (6-17).
The last four terms in Eq. (11-29) tend to be of similar magnitudes when the optical bandwidth \( \Delta \nu_{opt} \) is taken to be the optical bandwidth of the spontaneous emission noise, which covers a 30-nm spectrum (see Prob. 11-7). However, one generally uses a narrow optical filter at the receiver, so that \( \Delta \nu_{opt} \) is on the order of 125 GHz (a 1-nm spectral width at 1550 nm) or less. In that case, we can simplify Eq. (11-29) by examining the magnitudes of the various noise components. First, the thermal noise can generally be neglected when the amplifier gain is large enough. Furthermore, since the amplified signal power \( G P_{s, in} \) is much larger than the ASE noise power \( S_{ASE} \), the ASE--ASE beat noise given by Eq. (11-28) is significantly smaller than the signal--ASE beat noise. This observation reduces Eq. (11-26) to

\[
\sigma_{\text{shot}}^2 \approx 2q\Re G P_{s, in} B
\]

Using these results together with the expression for \( S_{ASE} \) from Eq. (11-24) yields the following approximate signal-to-noise ratio \( (S/N) \) at the photodetector output:

\[
\left( \frac{S}{N} \right)_{\text{out}} = \frac{\sigma_{\text{shot}}^2}{\sigma_{\text{total}}^2} \approx \frac{\Re^2 G^2 P_{s, in}^2}{2q B} \frac{G}{1 + 2\eta_0(G - 1)} \tag{11-31}
\]

where \( \eta \) is the quantum efficiency of the photodetector and, from Eq. (6-11), the mean-square input photocurrent is

\[
\left( \langle i_{\text{ph}}^2 \rangle \right) = \sigma_{\text{ph}}^2 = \Re G^2 P_{s, in}^2 \tag{11-32}
\]

Note that the term

\[
\left( \frac{S}{N} \right)_{\text{in}} = \frac{\Re G P_{s, in}}{2qB} \tag{11-33}
\]

in Eq. (11-31) is the signal-to-noise ratio of an ideal photodetector at the input to the optical amplifier. From Eq. (11-31) we can then find the noise figure of the optical amplifier, which is a measure of the \( S/N \) degradation experienced by a signal after passing through the amplifier. Using the standard definition of noise figure as the ratio between the \( S/N \) at the input and the \( S/N \) at the amplifier output, we have

\[
\text{Noise figure} = F = \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} = \frac{1 + 2\eta_0(G - 1)}{G} \tag{11-34}
\]

When \( G \) is large, this becomes \( 2\eta_0 \). A perfect amplifier would have \( \eta_0 = 1 \), yielding a noise figure of 2 (or 3 dB), assuming \( \eta = 1 \). That is, using an ideal receiver with a perfect amplifier would degrade the \( S/N \) by a factor of 2. In a real EDFA, for example, \( \eta_0 \) is around 2, so the input \( S/N \) gets reduced by a factor of about 4.

**Example 11-5.** Figure 11-10 shows measured values of the noise figure for an EDFA under gain saturation for both codirectional and counterdirectional pumping.\(^{26}\) The pump wavelength was 1480 nm and the signal wavelength was 1550 nm with an input power to the amplifier of \(-66 \text{ dBm}\). Under small-signal conditions, the codirectional pumping noise was about 5.5 dB, which included a 1.5-dB input coupling loss. The noise figure of the optical amplifier itself was thus 4 dB, compared with the theoretical minimum of 3 dB with complete population inversion. The noise figure in the counterdirectional pumping case was about 1 dB higher.

### 11.5 SYSTEM APPLICATIONS

In designing an optical fiber link that requires optical amplifiers, there are three possible locations where the amplifiers can be placed, as shown in Fig. 11-1. Although the physical amplification process is the same in all three configurations, the various uses require operation of the device over different input power ranges. This, in turn, implies use of different amplifier gains. The complete analysis of the signal-to-noise ratios, taking into account factors such as detailed photon statistics, and discrete amplifier configurations, are fairly involved. Desurvire\(^{16}\) gives an extensive treatment for readers who need more detail. Here, we will look at simple conceptual analysis and present generic operational values for the three possible locations of EDFAs in an optical link.

#### 11.5.1 Power Amplifiers

For the power amplifier, the input power is high, since the device immediately follows an optical transmitter. High pump powers are normally required for this application.\(^{26}\) The amplifier inputs are generally \(-8 \text{ dBm}\) or greater, and the power amplifier gain must be greater than 5 dB in order for it to be more advantageous than using a preamplifier at the receiver.